Ontology-Driven Conceptual Modeling

nemo

Giancarlo Guizzardi
Computer Science Department
Federal University of Espírito Santo (UFES), Brazil

IAOA Summer School
Trento, Italy
Day 4

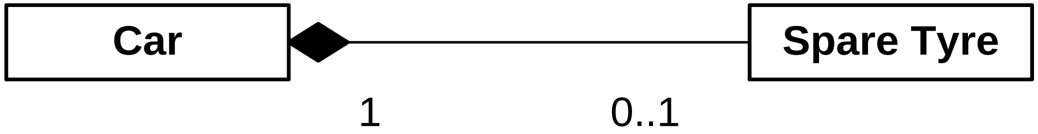
Modal Logics

- For this presentation, I will use the simplest system of quantified alethic model logics (QS5);
- The accessability relation is considered to be universal (all worlds are equally accessible);
- Worlds are taken to be maximal state of affairs;
- In order to simplify the formulae, I will sometimes make use of restricted quantification:
  - $(\forall S,x)\ A \equiv (\forall x\ S(x) \rightarrow A)$
  - $(\exists S,x)\ A \equiv (\exists x\ S(x) \land A)$

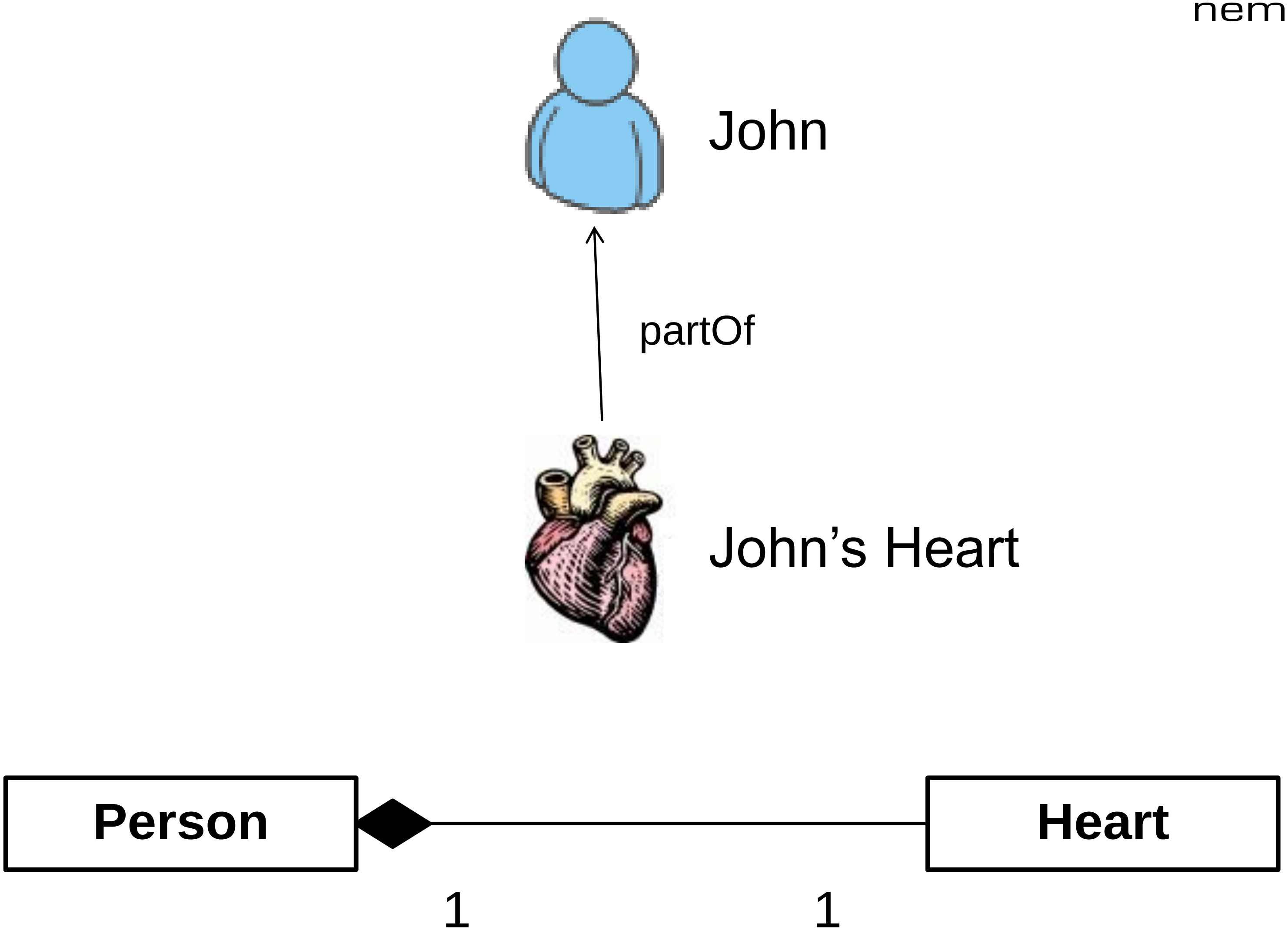
OPTIONAL, MANDATORY, IMMUTABLE AND ESSENTIAL PARTHOOD

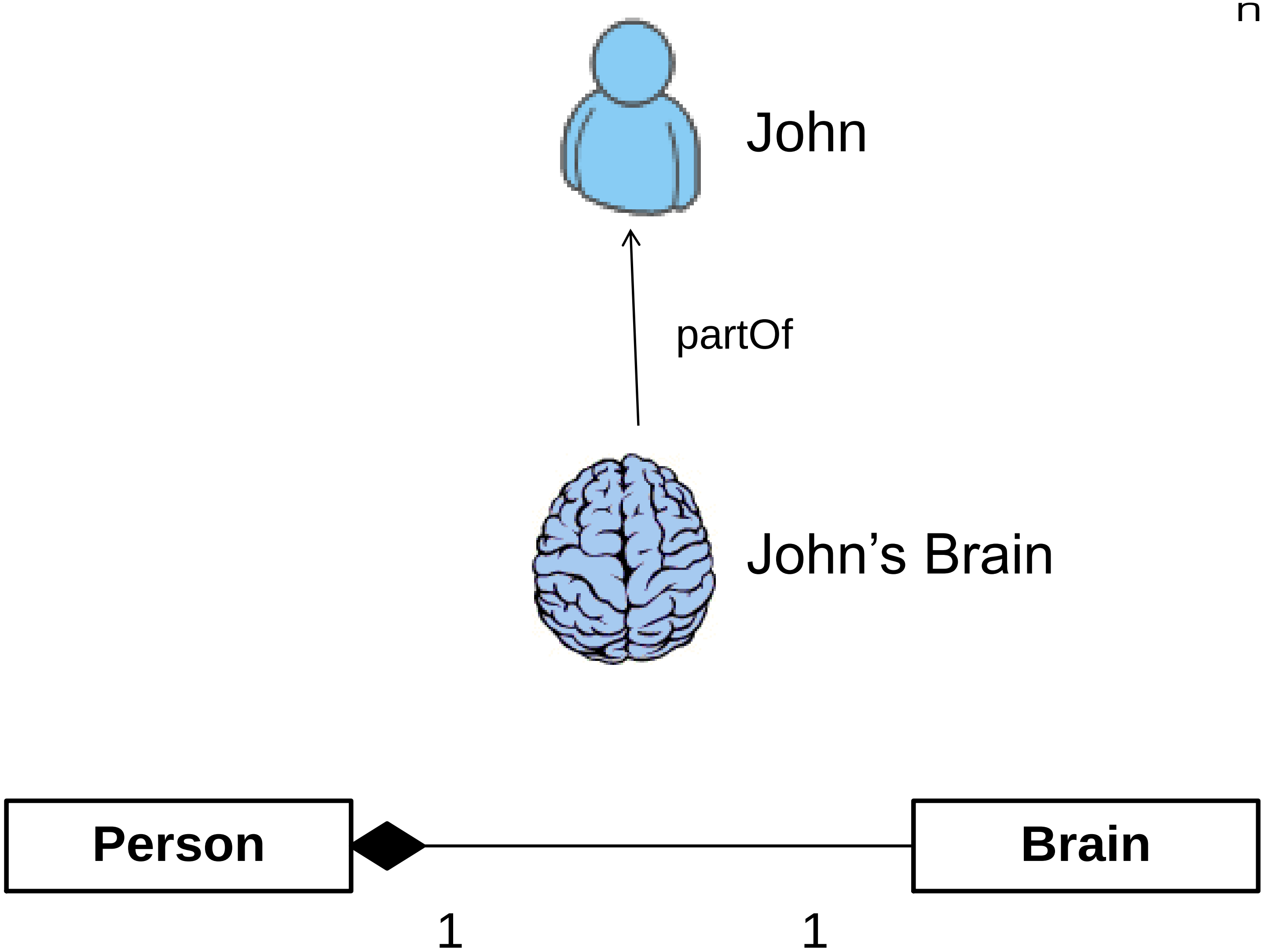
Optional Parts

- Optional Parts are parts that the whole can lack without having any effect on its classification or identity



- Contrast it with non-optional parts:


John
partOf
John’s Heart
Person
Heart
1
1


John
partOf
John’s Brain
Person
Brain
1
1

Different Types of Dependence Relations

- The two relations just described reflect two different types of ontological dependence relations

Generic Dependence

- An individual *y* is *generic dependent* of a type T if for y to exists it requires an instance of T to exists as well

$$\mathbf{GD(y,T) =_{def} \Box(\varepsilon(y) \rightarrow \exists x\ T(x) \wedge \varepsilon(x))}$$

Generic Dependence

- An individual *y* is *generic dependent* of a type T if for y to exists it requires an instance of T to exists as well

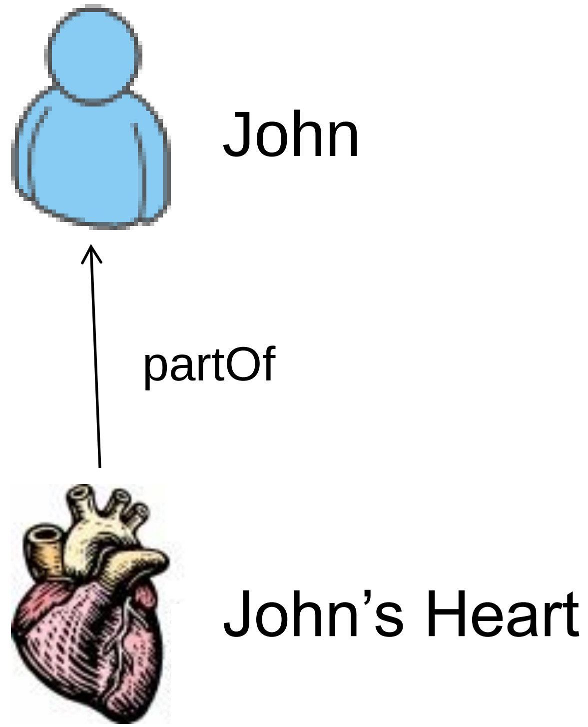
$$\mathbf{GD(y,T) =_{def} \Box(\varepsilon(y) \rightarrow \exists x\ T(x) \wedge \varepsilon(x))}$$

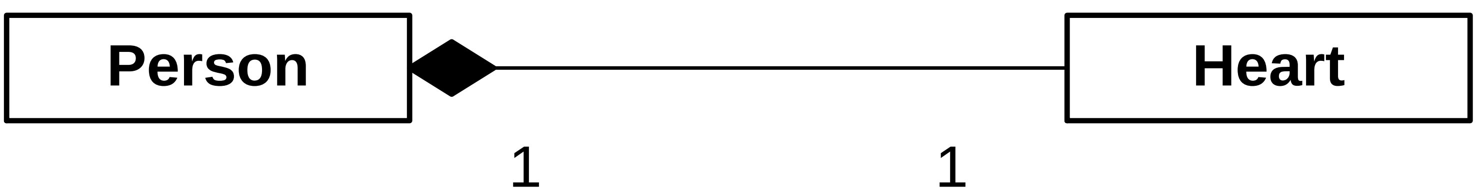
represents existence

Mandatory Parthood

- An individual *x* is a mandatory part of another individual *y* if *y* is generically dependent of an type T that *x* instantiates, and y has, necessarily, as a part an instance of T*:*

$$\mathbf{MP(T,y) =_{def} \Box(\varepsilon(y) \rightarrow (\exists x\ T(x) \wedge (x < y))}$$





$$\Box(\forall x\ Person(x) \rightarrow (\Box(\varepsilon(x) \rightarrow \exists! y\ Heart(y) \wedge (y < x))))$$

Existential Dependence

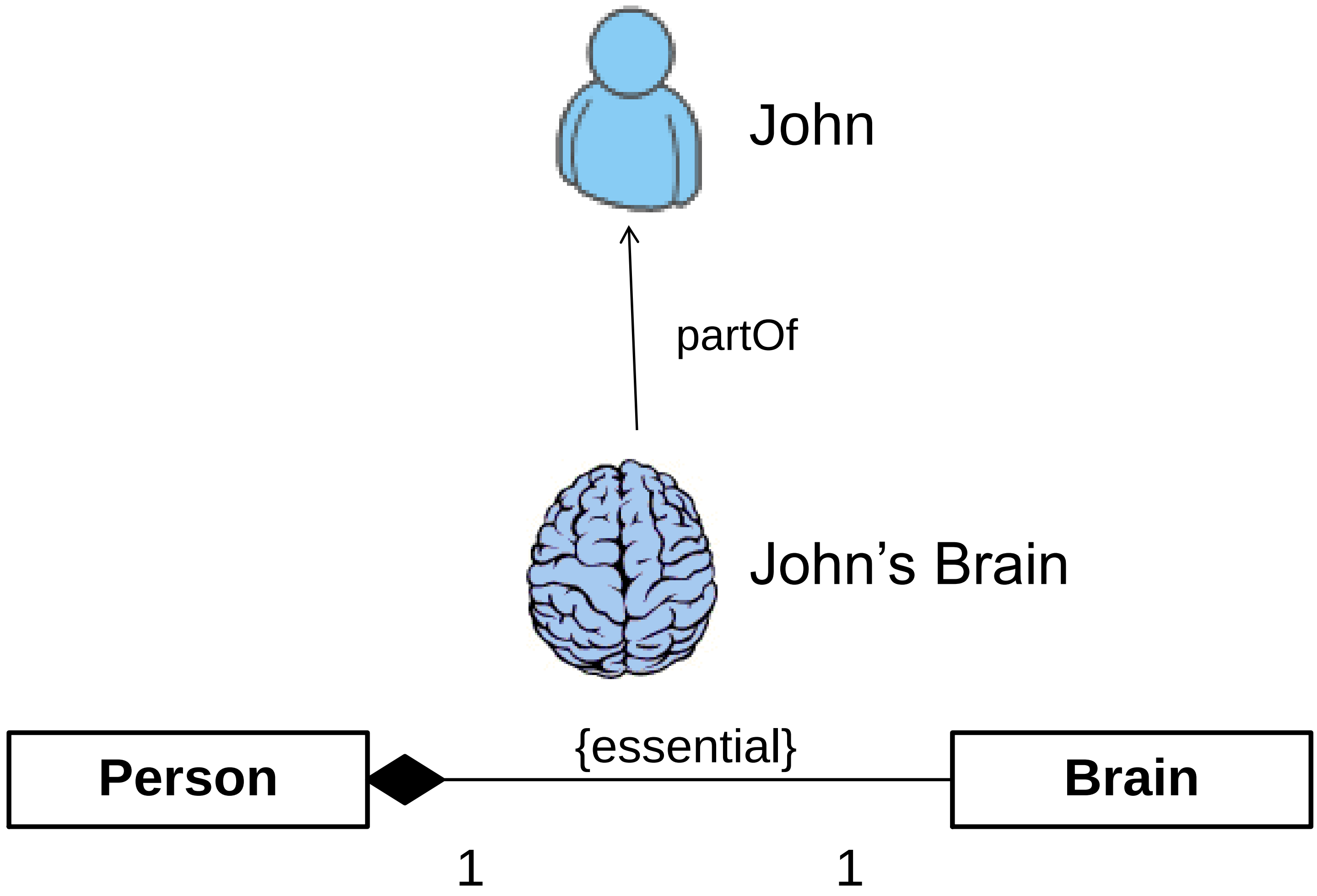
- We have that an individual *x* is *existentially dependent* on another individual *y* (symbolized as *ed(x,y)*) if, in order to exist, x requires that (one specific) y exists as well

$$\mathbf{ed(x,y) =_{def} \Box(\varepsilon(x) \rightarrow \varepsilon(y))}$$

Essential Parthood

- An individual *x* is an essential part of another individual *y* if *y* is existentially dependent on *x* and *x* is, necessarily, a part of *y:*

$$\mathbf{EP(x,y) =_{def} \Box(\varepsilon(y) \rightarrow (x \leq y))}$$



$$\Box(\forall x\ \mathbf{Person}(x) \rightarrow (\exists! y\ \mathbf{Brain}(y) \wedge \Box(\varepsilon(x) \rightarrow (y < x))))$$

Contrast Mandatory vs. Essential Parthood

Mandatory Part

$$\Box(\forall x\ \mathbf{Person}(x) \rightarrow (\Box(\varepsilon(x) \rightarrow \exists! y\ \mathbf{Heart}(y)) \wedge (y < x))))$$

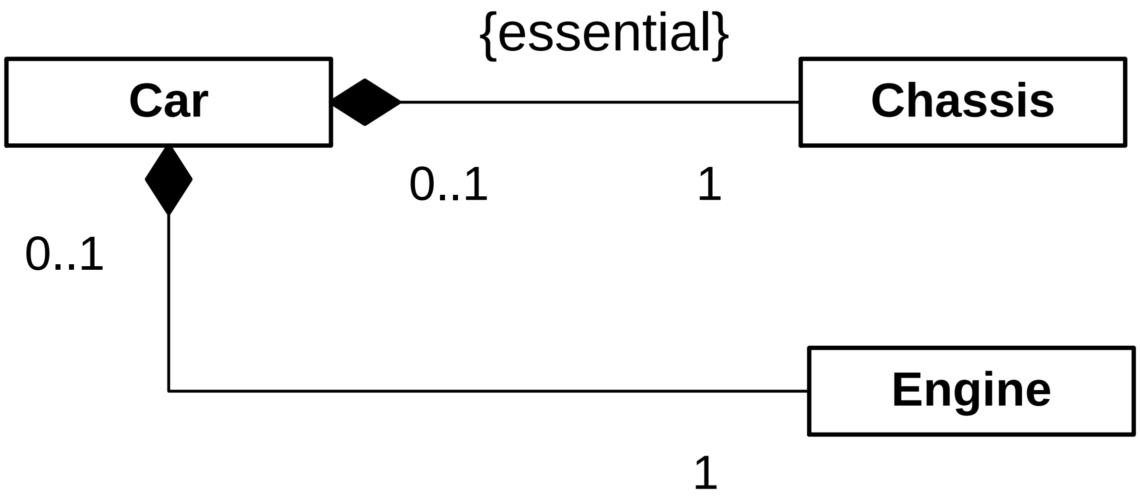
Consider $\Box$ as an iterator over a set of worlds
The instance of heart can change from world to world

Essential Part

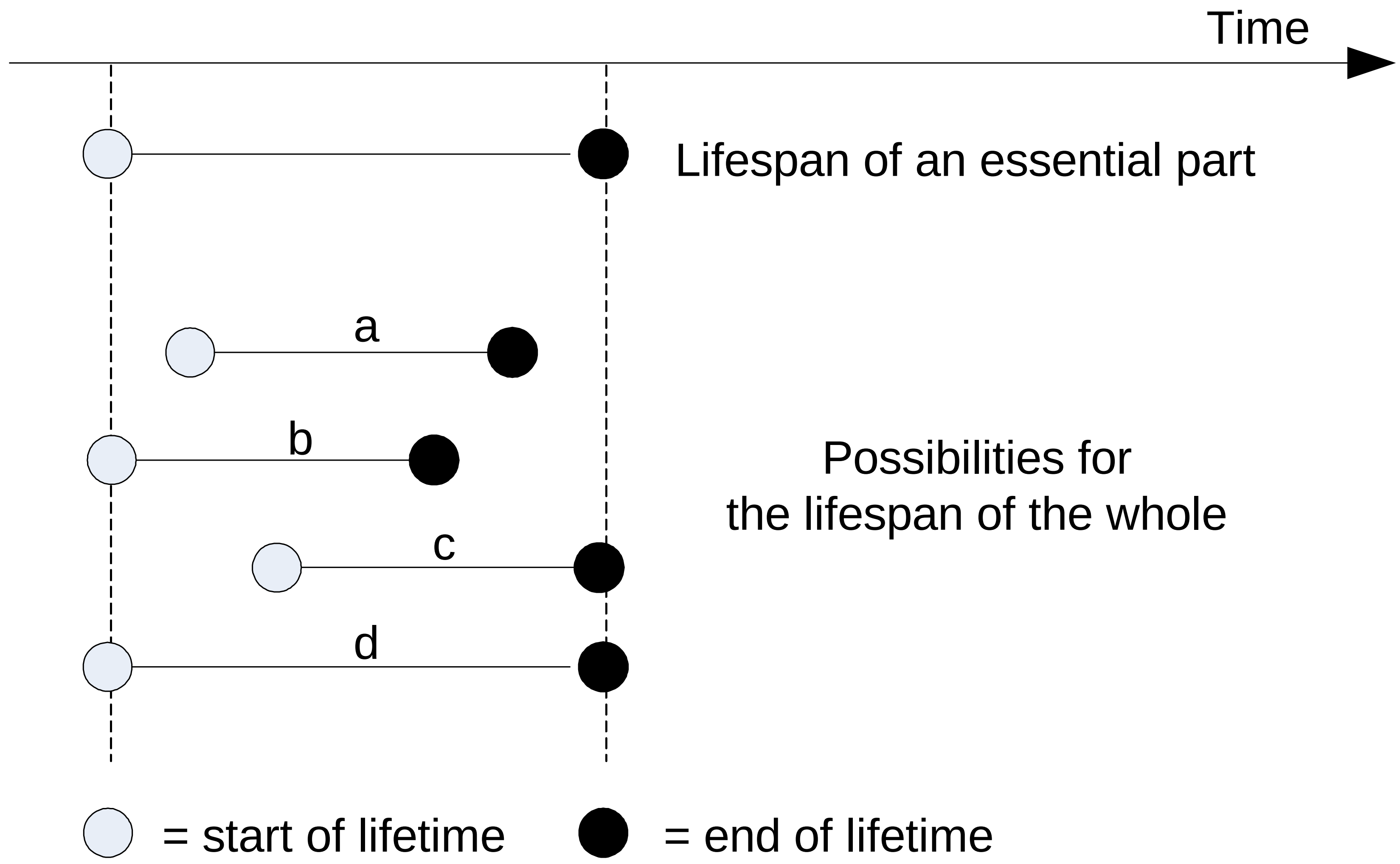
$$\Box(\forall x\ \mathbf{Person}\ (x) \rightarrow (\exists! y\ \mathbf{Brain}(y)) \wedge \Box(\varepsilon(x) \rightarrow (y < x))))$$

The instance of Brain is selected before $\Box$ starts
Iterating from world to world, i.e., the instance of
Brain is fixed and cannot change!

Essential vs. Mandatory Part



Essential Parts



Extensional Individuals

- Now, one can easily define the notion of an extensional individual (from the extensional mereology) using the notion of Essential Parts

$$\mathbf{Ext(y) =_{def} \Box(\forall x\ (x < y) \rightarrow EP(x,y))}$$

Now from the part to the whole…

Inseparable Parthood

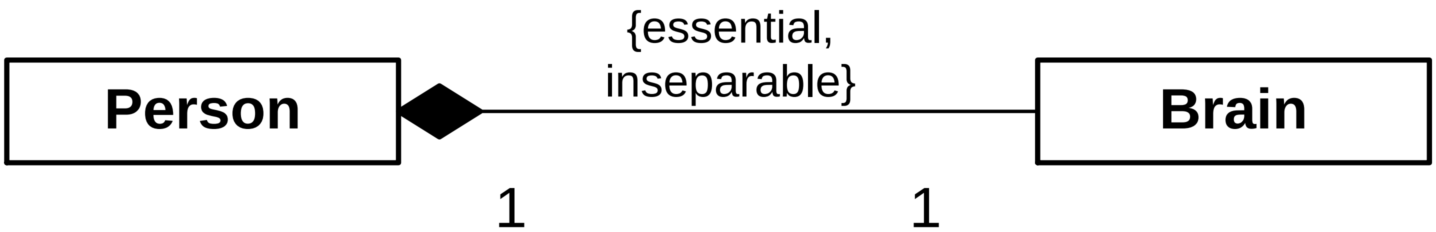
- An individual *x* is an inseparable part of another individual *y* if *x* is existentially dependent on *y*, and *x* is, necessarily, a part of *y:*

$$\mathbf{IP(x,y) =_{def} \Box(\varepsilon(x) \rightarrow (x \leq y))}$$

Inseparable Parthood

- An individual *x* is an inseparable part of another individual *y* if *x* is existentially dependent on *y*, and *x* is, necessarily, a part of *y:*

$$\mathbf{IP(x,y) =_{def} \Box(\varepsilon(x) \rightarrow (x \leq y))}$$



$$\mathbf{\Box(\forall x\ Brain(x) \rightarrow (\exists! y\ Person(y) \land \Box(\varepsilon(x) \rightarrow (x < y))))}$$

Mandatory Wholes

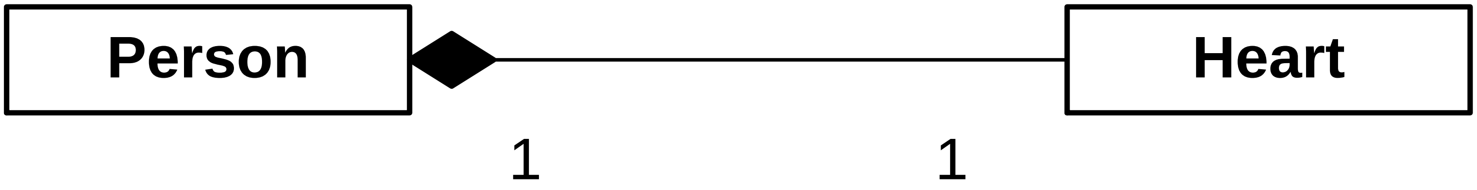
- An individual *y* is a mandatory whole for another individual *x* if, *x* is generically dependent on a type T that *y* instantiates, and *x* is, necessarily, part of an individual instantiating T:

$$\mathbf{MW(T,x) =_{def} \square(\varepsilon(x) \rightarrow (\exists y\ T(y) \wedge (x < y)))}$$

Mandatory Wholes

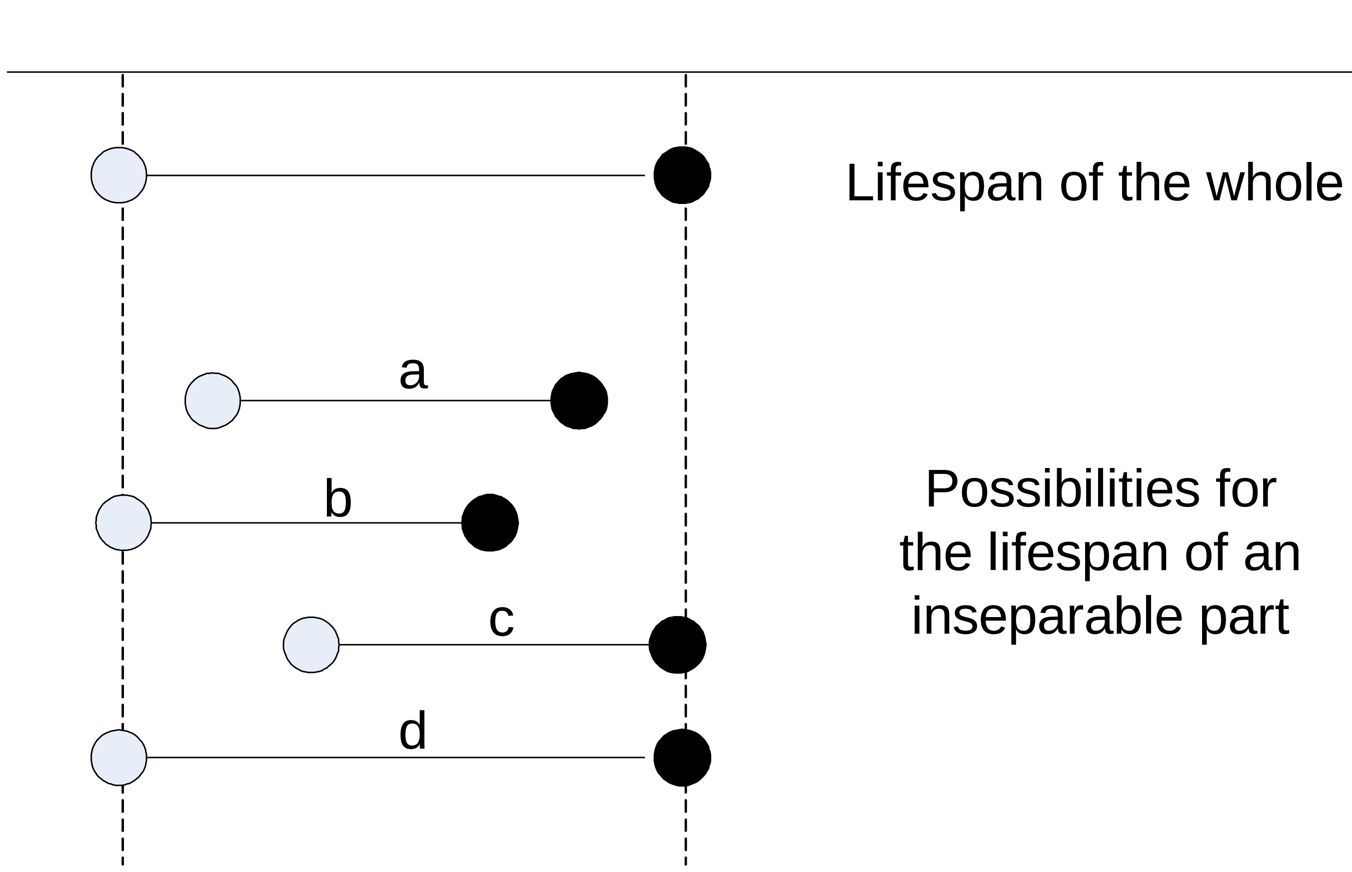
- An individual *y* is a mandatory whole for another individual *x* if, *x* is generically dependent on a type T that *y* instantiates, and *x* is, necessarily, part of an individual instantiating T:

$$\mathbf{MW(T,x) =_{def} \Box(\varepsilon(x) \rightarrow (\exists y\ T(y) \land (x < y)))}$$



$$\mathbf{\Box(\forall x\ Heart(x) \rightarrow (\Box(\varepsilon(x) \rightarrow \exists! y\ Person(y) \land (x < y))))}$$

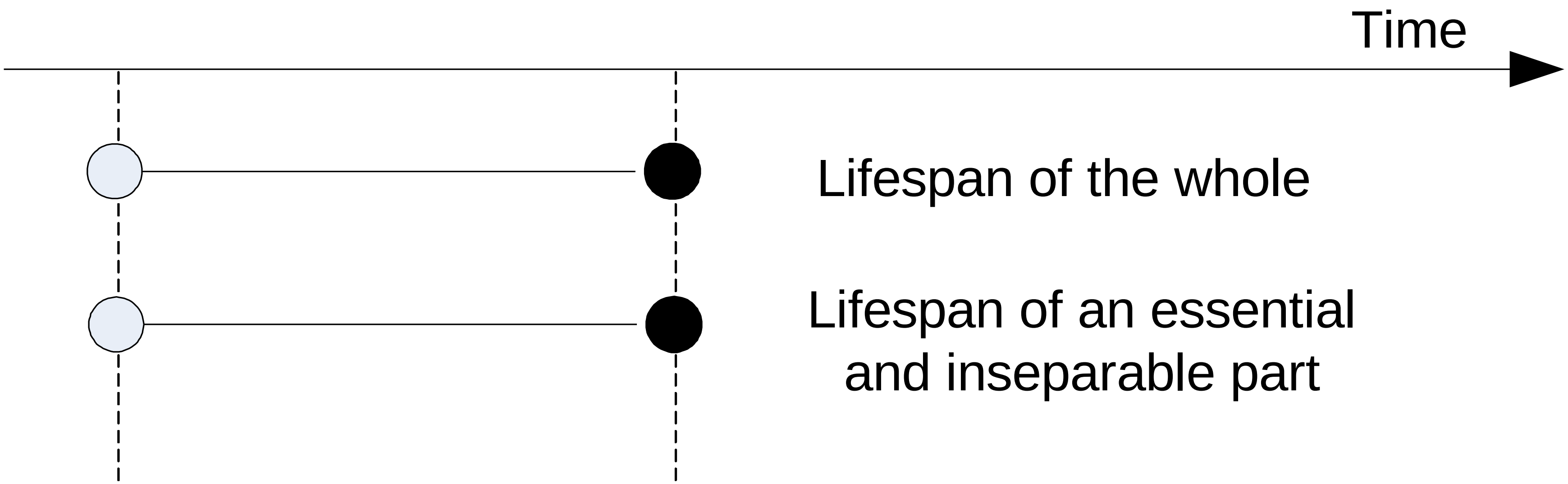
Inseparable Parts



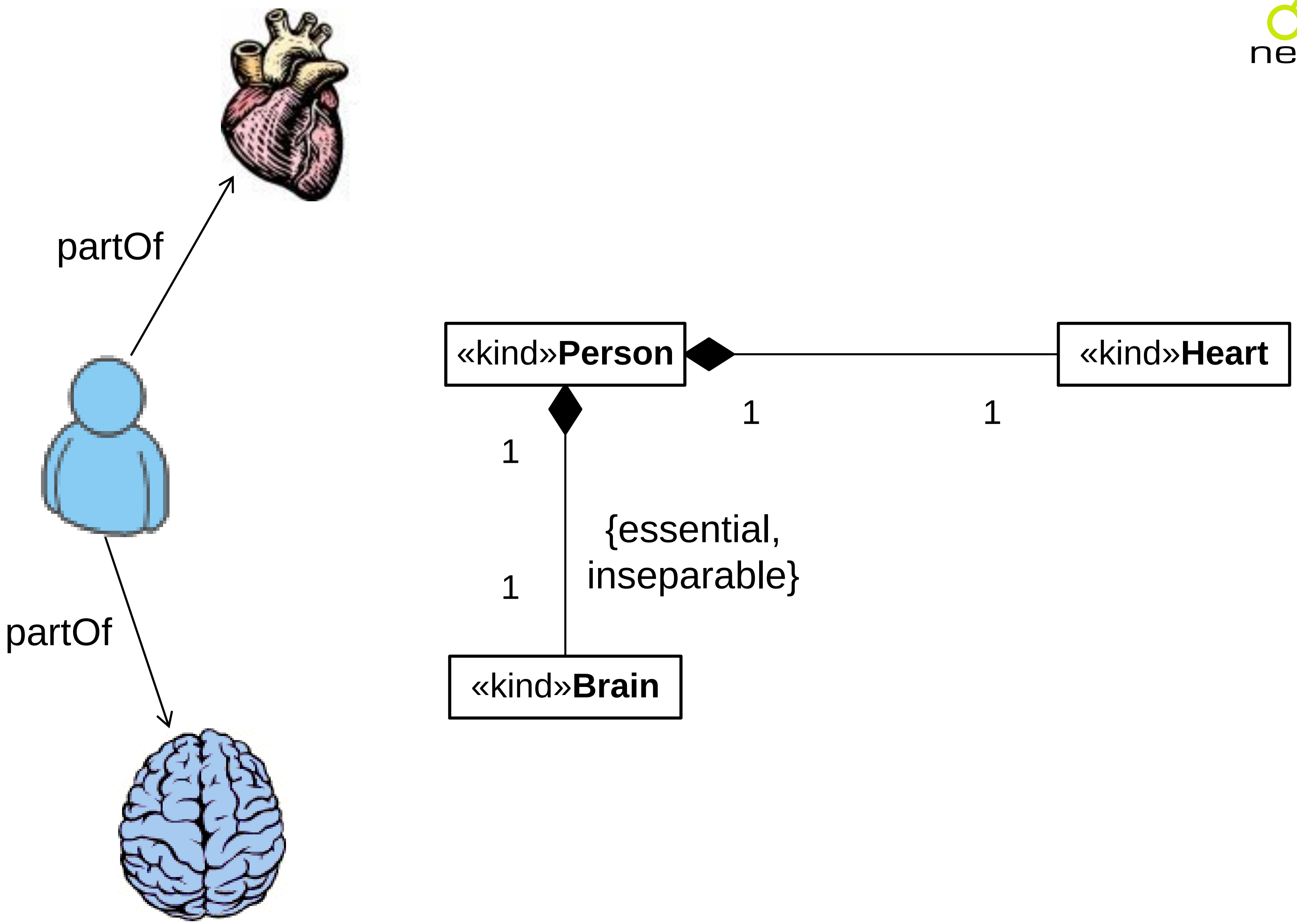
Essentiality and Inseparability

- Essentiality does not imply inseparability:
  - Think about a *Collected Works* publication of some authors. It is defined by that specific extensional collection of papers, but the papers could exist prior to and outlive the collection
- Inseparability does not imply Essentiality :
  - A whole in this table is an inseparable part of it, but not an essential part of the table

Essential and Inseparable Parts

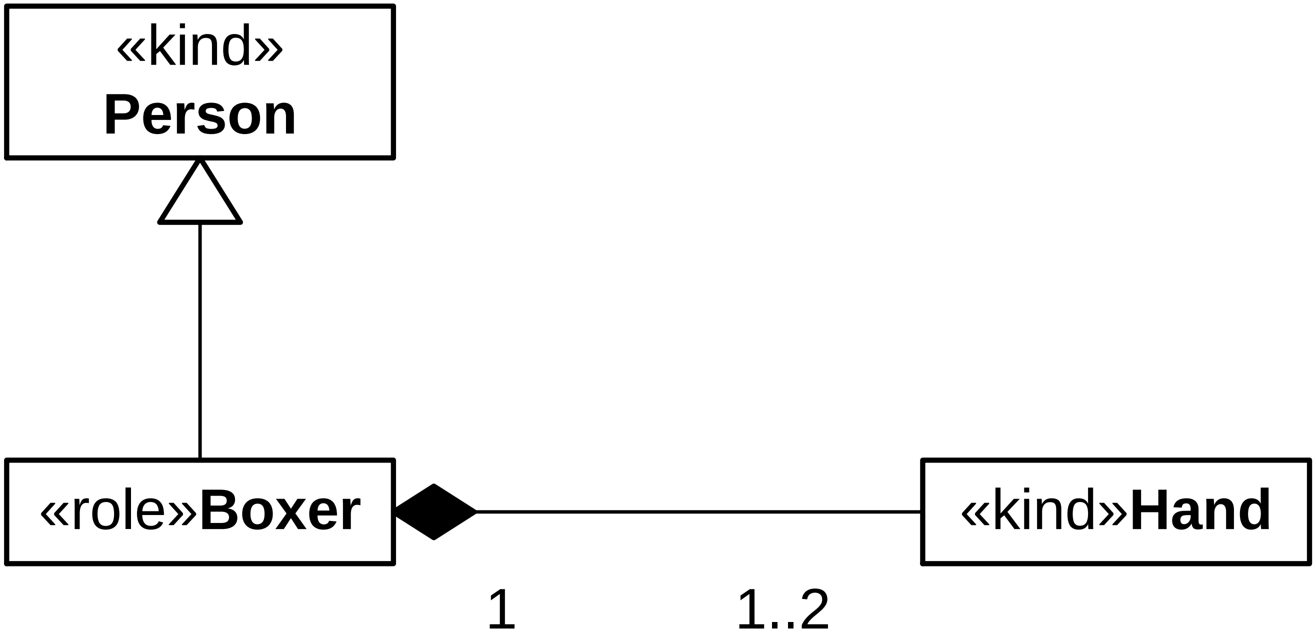


As we will see later Essentiality does not imply unshareability...
Unshareability also does not imply Essentiality!

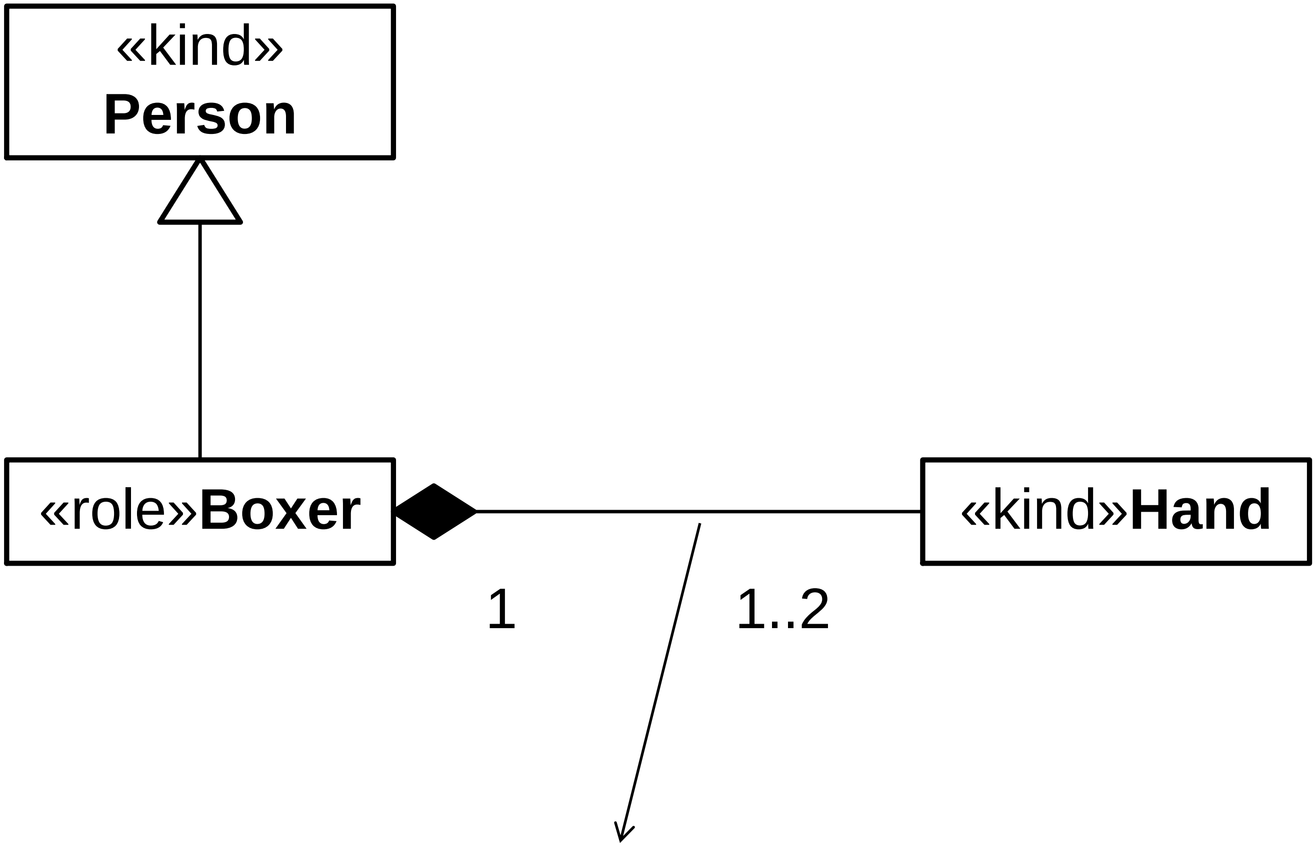

partOf
partOf
«kind»Person
«kind»Heart
«kind»Brain
1
1
1
1
{essential,
inseparable}

Parts of Anti-Rigid Object Types

- “every boxer must have a hand”
- “every biker must have a leg”

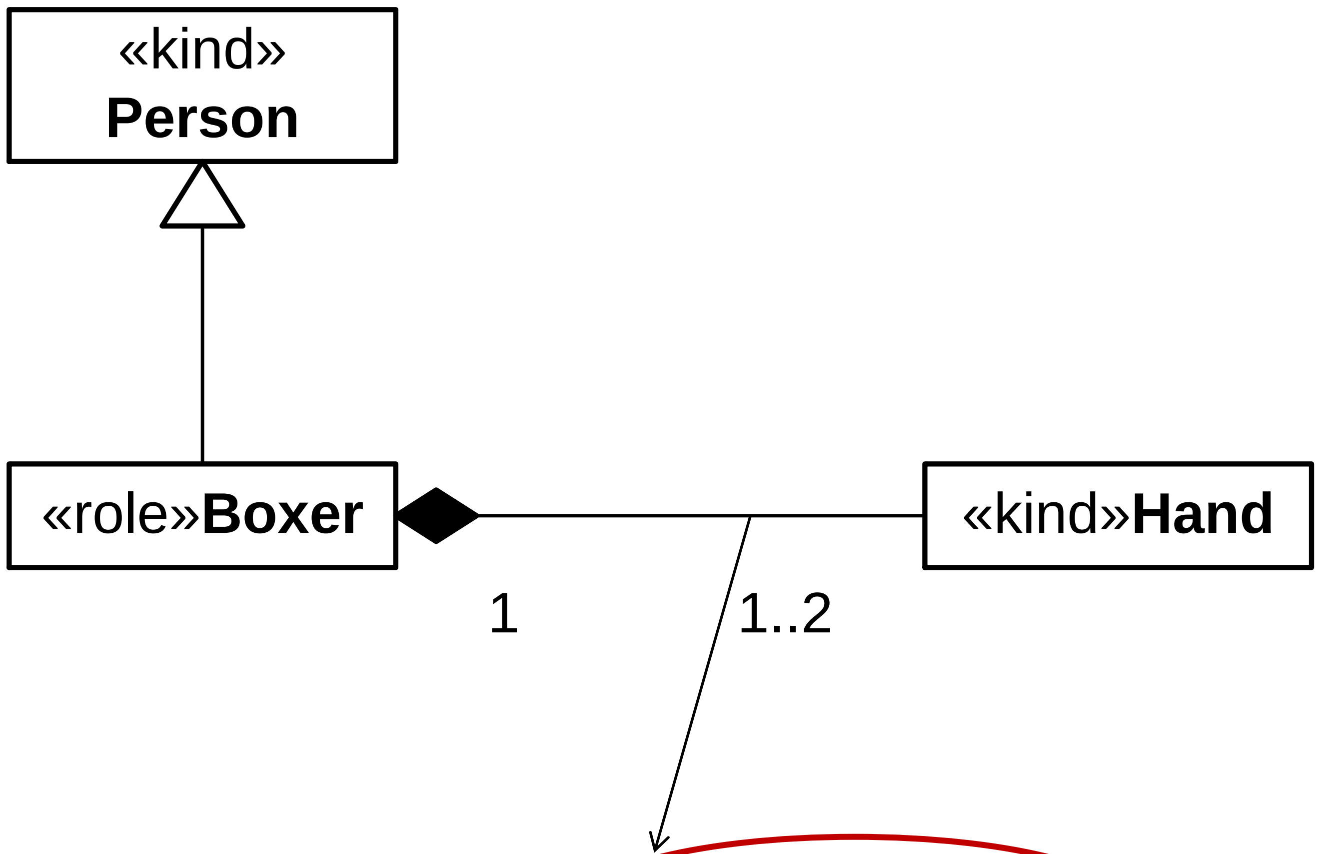

«kind»
Person
«role»Boxer
«kind»Hand
1
1..2

Mandatory Part?



$$\Box(\forall x\ \mathbf{Boxer}(x) \rightarrow (\Box(\varepsilon(x) \rightarrow \exists y\ \mathbf{Hand}(y) \wedge (y < x))))$$

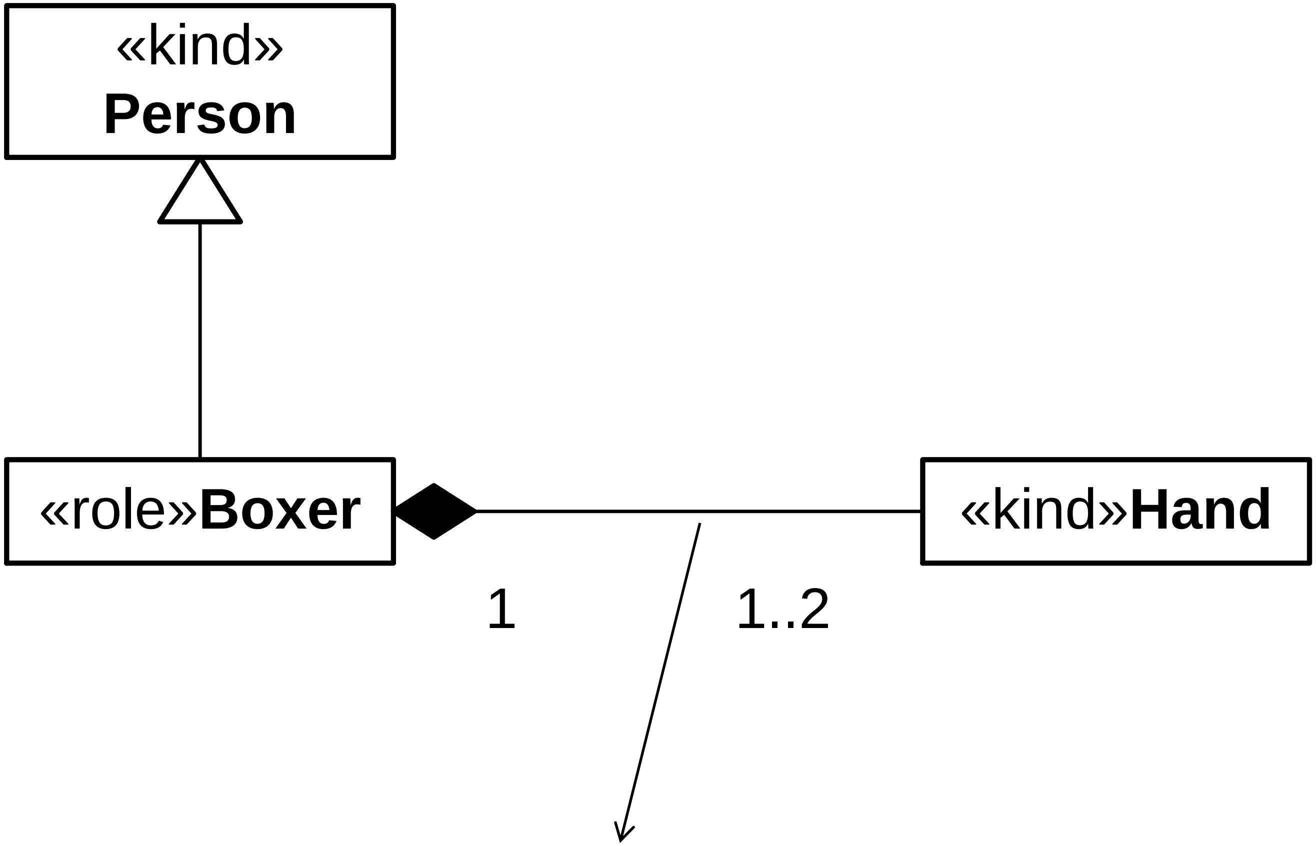
Mandatory Part?



$$\Box(\forall x\ \mathbf{Boxer}(x) \rightarrow (\Box(\varepsilon(x) \rightarrow \exists y\ \mathbf{Hand}(y)) \wedge (y < x))))$$

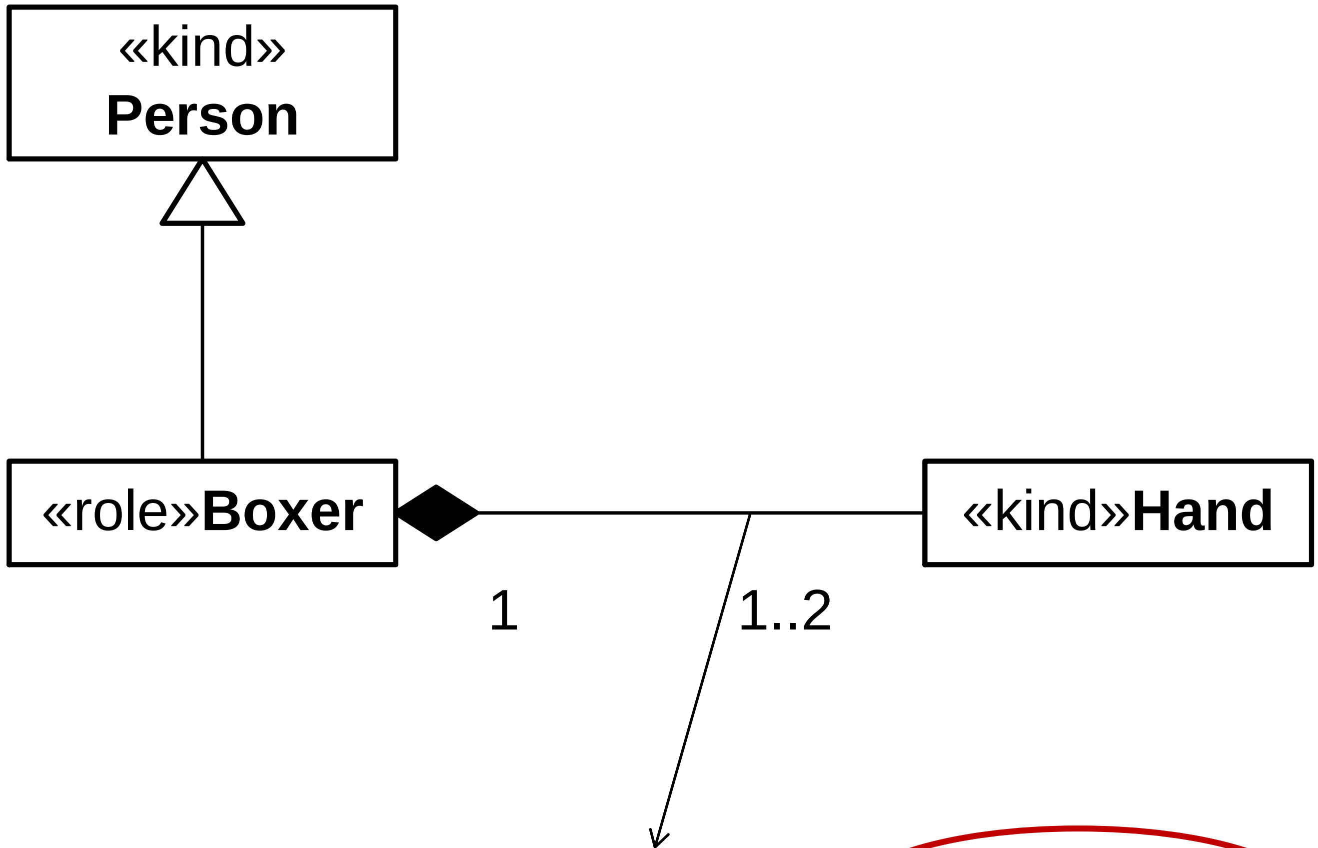
The instance of hand can change from world to world?

Mandatory Part?



$$\Box(\forall x\ Boxer(x) \rightarrow (\exists y\ hand(y)\ \wedge \Box(\varepsilon(x) \rightarrow (y < x))))$$

Essential Part?



$$\Box(\forall x\ \mathbf{Boxer}(x) \rightarrow (\exists y\ \mathbf{hand}(y)\quad \Box(\varepsilon(x) \rightarrow (y < x))))$$

This implies that the boxer must have that hand in every possible situation. Is it true?

De Re/De Dicto Modalities

(i) The queen of the Netherlands is necessarily queen;

(ii) The number of planets in the solar system is necessarily even.

Sentence (i)

- The queen of the Netherlands is necessarily queen:

$\forall x$ QueenOfTheNetherlands(x) $\rightarrow \Box$(Queen(x)) ← DE RE

$\Box(\forall x$ QueenOfTheNetherlands(x) $\rightarrow$ Queen(x)) ← DE DICTO

Sentence (i)

- The queen of the Netherlands is necessarily queen:

$\forall x\ QueenOfTheNetherlands(x) \rightarrow \Box(Queen(x))$ ← DE RE

$\Box(\forall x\ QueenOfTheNetherlands(x) \rightarrow Queen(x))$ ← DE DICTO

Sentence (ii)

- The number of planets in the solar system is necessarily even:

$\forall x \; NumberOfPlanets(x) \rightarrow \Box(Even(x)))$ ← DE RE

$\Box(\forall x \; NumberOfPlanets(x) \rightarrow Even(x)))$ ← DE DICTO

Sentence (ii)

- The number of planets in the solar system is necessarily even:

$\forall x\ NumberOfPlanets(x) \rightarrow \Box(Even(x)))$ ← DE RE

$\Box(\forall x\ NumberOfPlanets(x) \rightarrow Even(x)))$ ← ✗ DE DICTO

The Boxer Example

"every boxer must have a hand"

"If someone is a boxer than he has at least a hand in every possible circumstance" ← DE RE

$$\Box((\forall x\ \mathbf{Boxer}(x) \rightarrow \exists y\ \mathbf{Hand}\ (y) \wedge \Box(\varepsilon(x) \rightarrow (y < x)))$$

$$\Box((\forall x\ \mathbf{Boxer}(x) \rightarrow \Box(\varepsilon(x) \rightarrow \exists y\ \mathbf{Hand}(y) \wedge (y < x)))$$

"In any circumstance, whoever is boxer has at least one hand" ← DE DICTO

The Boxer Example

“every boxer must have a hand”

“If someone is a boxer than he has at least a hand in every possible circumstance” ← DE RE

$$\Box((\forall x\ \mathrm{Boxer}(x) \rightarrow \exists y\ \mathrm{Hand}(y) \wedge \Box(\varepsilon(x) \rightarrow (y < x)))$$

$$\Box((\forall x\ \mathrm{Boxer}(x) \rightarrow \Box(\varepsilon(x) \rightarrow \exists y\ \mathrm{Hand}(y) \wedge (y < x)))$$

“In any circumstance, whoever is boxer has at least one hand” ← DE DICTO

The Boxer Example

“every boxer must have a hand”

“If someone is a boxer than he has at least a hand in every possible circumstance” ← DE RE

$\Box(\forall x\ Boxer(x) \rightarrow \exists y\ Hand(y) \wedge \Box(\varepsilon(x) \rightarrow (y < x)))$

$\Box(\forall x\ Boxer(x) \rightarrow \Box(\varepsilon(x) \rightarrow \exists y\ Hand(y) \wedge (y < x)))$

“In any circumstance, whoever is boxer has at least one hand” ← DE DICTO

$\Box(\forall x\ Boxer(x) \rightarrow \exists y\ Hand(y) \wedge \Box(\varepsilon(x) \wedge Boxer(x) \rightarrow (y < x)))$

The Boxer Example

nemo

“every boxer must have a hand”

“If someone is a boxer than he has at least a hand in every possible circumstance” ← DE RE

$$\Box(\forall x\ Boxer(x) \rightarrow \exists y\ Hand(y) \wedge \Box(\varepsilon(x) \rightarrow (y < x)))$$

$$\Box(\forall x\ Boxer(x) \rightarrow \Box(\varepsilon(x) \rightarrow \exists y\ Hand(y) \wedge (y < x)))$$

“In any circumstance, whoever is boxer has at least one hand” ← DE DICTO

$$\Box(\forall x\ Boxer(x) \rightarrow \exists y\ Hand(y) \wedge \Box(\varepsilon(x) \wedge Boxer(x)) \rightarrow (y < x)))$$

On one hand, the object is fixed

.On the other hand, the parthood relation is only required to take place in those worlds in which the person is a Boxer

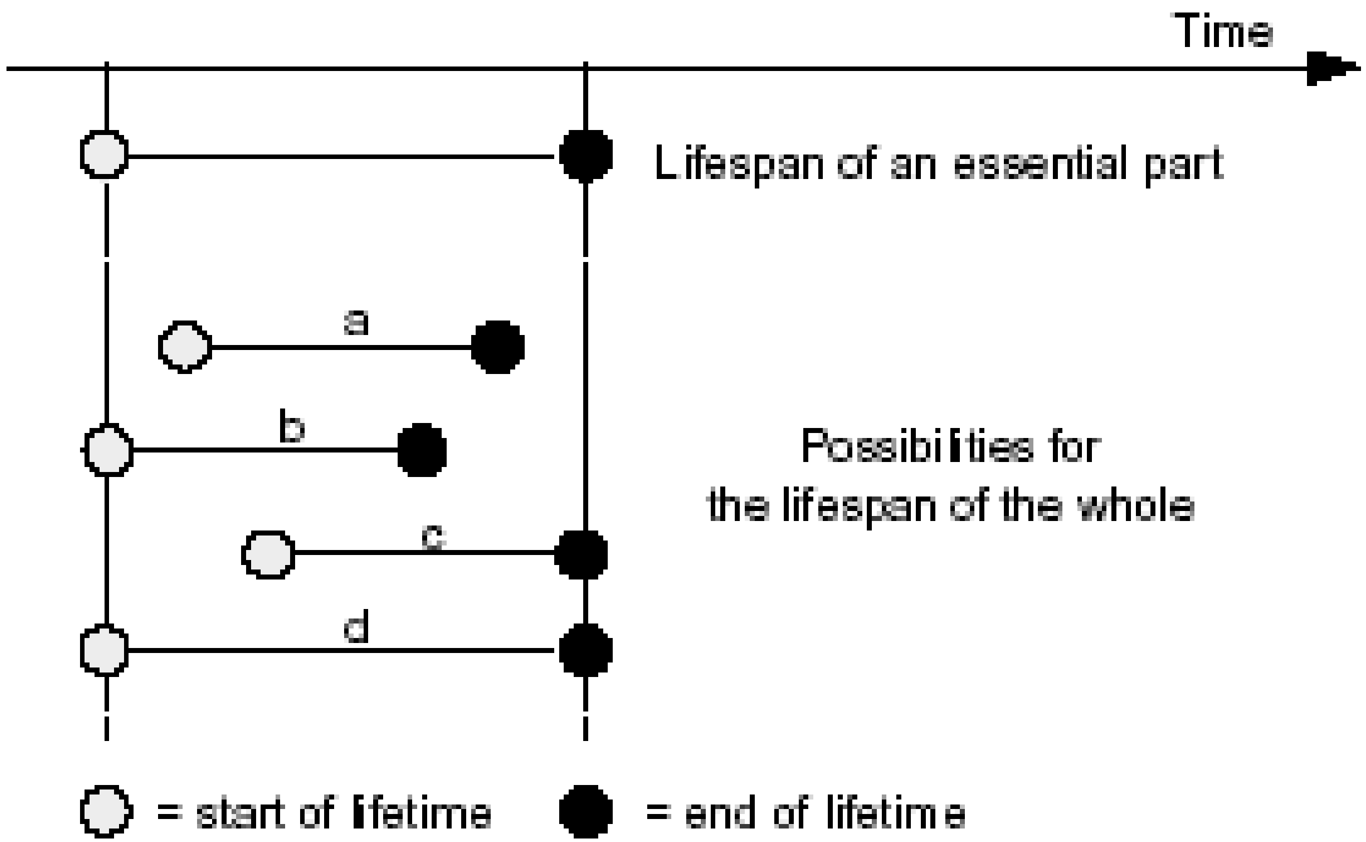
Further Distinctions among Part-Whole relations

(i) specific dependence with *de re* modality: Essential Parts

(ii) generic dependence with *de re* modality: Mandatory parts

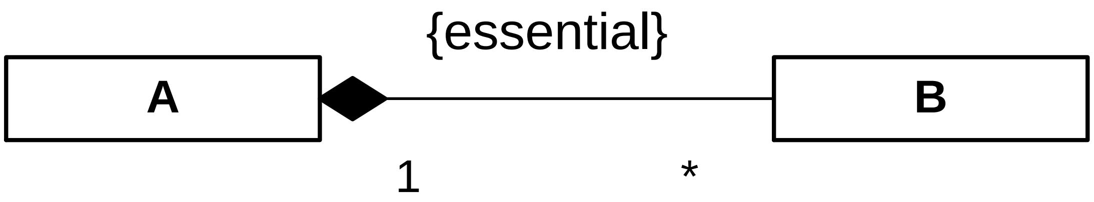
(iii) specific dependence with *de dicto* modality: Immutable parts

Lifetime Dependency (Essential Parts)



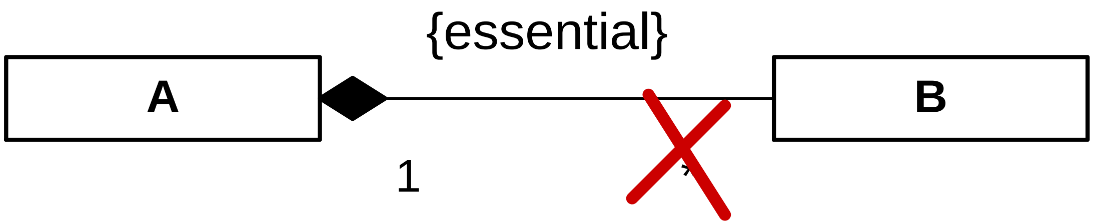
Notice that

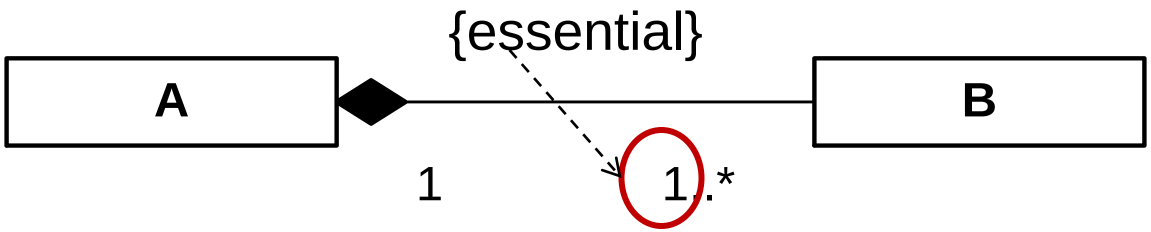
1. Only Rigid Types can be connected to Essential Parts
2. Essential implies Mandatory
   – If you need to have a specific part X in every possible situation then you need to have a part of type T (where T is the type of X) in every possible situation



Notice that

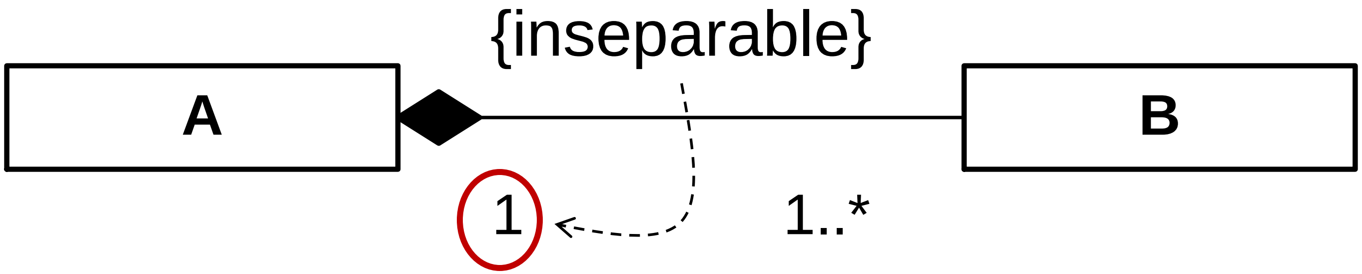
1. Only Rigid Types can be connected to Essential Parts
2. Essential implies Mandatory
   – If you need to have a specific part X in every possible situation then you need to have a part of type T (where T is the type of X) in every possible situation





Notice that

3. Inseparable implies Mandatory Whole
   – If you need to be part of a specific whole X in every possible situation then you need to be part of an instance of type T (where T is the type of X) in every possible situation



Notice that

4. Essential implies Immutable Part
5. Inseparable implies Immutable Whole

The De Dicto equivalent of De Re formulae

Essential Part (DE RE)

$$\Box(\forall x\ \mathbf{Person}(x) \rightarrow \exists! y\ \mathbf{Brain}(y) \wedge$$
$$\Box(\varepsilon(x) \rightarrow (y < x)))$$

Inseparable Part (DE DICTO)

$$\Box(\forall x\ \mathbf{Person}(x) \rightarrow \exists! y\ \mathbf{Brain}(y) \wedge$$
$$\Box(\varepsilon(x) \wedge \mathbf{Person}(x) \rightarrow (y < x)))$$

The De Dicto equivalent of De Re formulae

nemo

Essential Part (DE RE)

$\square(\forall x\ Person(x) \rightarrow \exists! y\ Brain(y) \wedge$
$\square(\varepsilon(x) \rightarrow (y < x)))$

Essential Part (DE DICTO)

$\square(\forall x\ Person(x) \rightarrow \exists! y\ Brain(y) \wedge$
$\square(\varepsilon(x) \wedge Person(x) \rightarrow (y < x)))$

Notice that this identical to the definition of Immutable Parts
If Person is Rigid then this is always true! In other words,
an Essential Part is immutable part defined for a Rigid Type

The De Dicto equivalent of De Re formulae

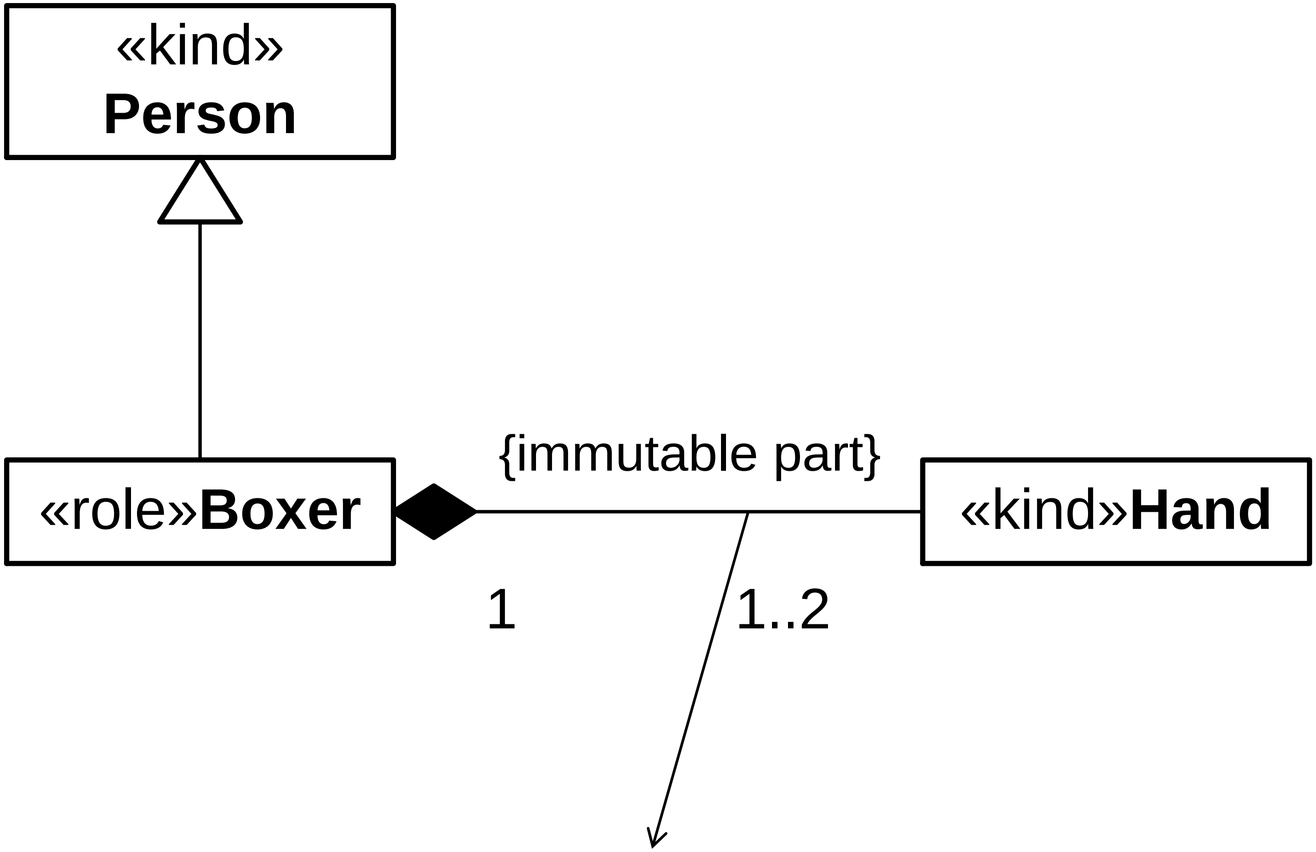
Mandatory Part (DE RE)

$\Box(\forall x \; \mathbf{Person}\,(x) \rightarrow \Box(\varepsilon(x) \rightarrow \exists! y \; \mathbf{Heart}(y) \wedge (y < x)))$

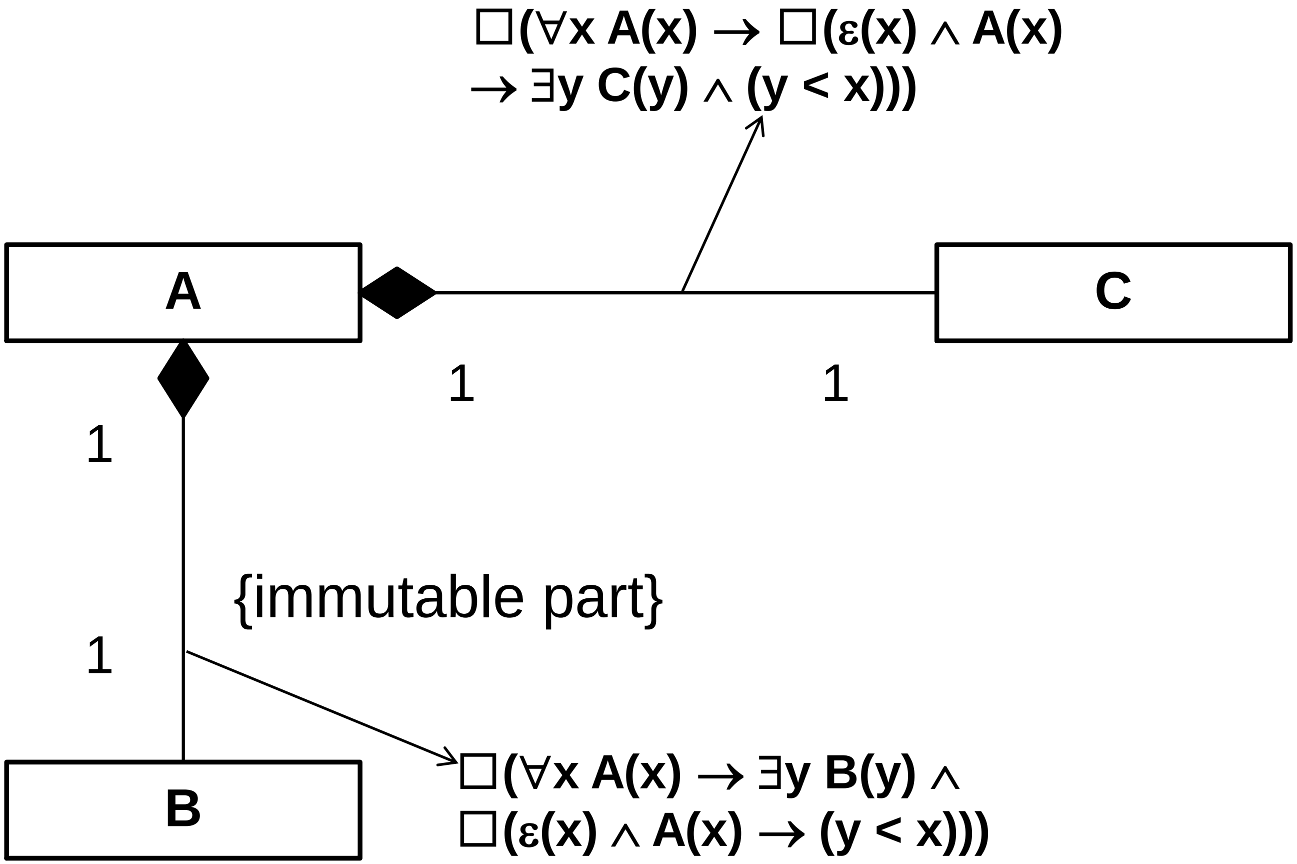
Mandatory Part (DE DICTO)

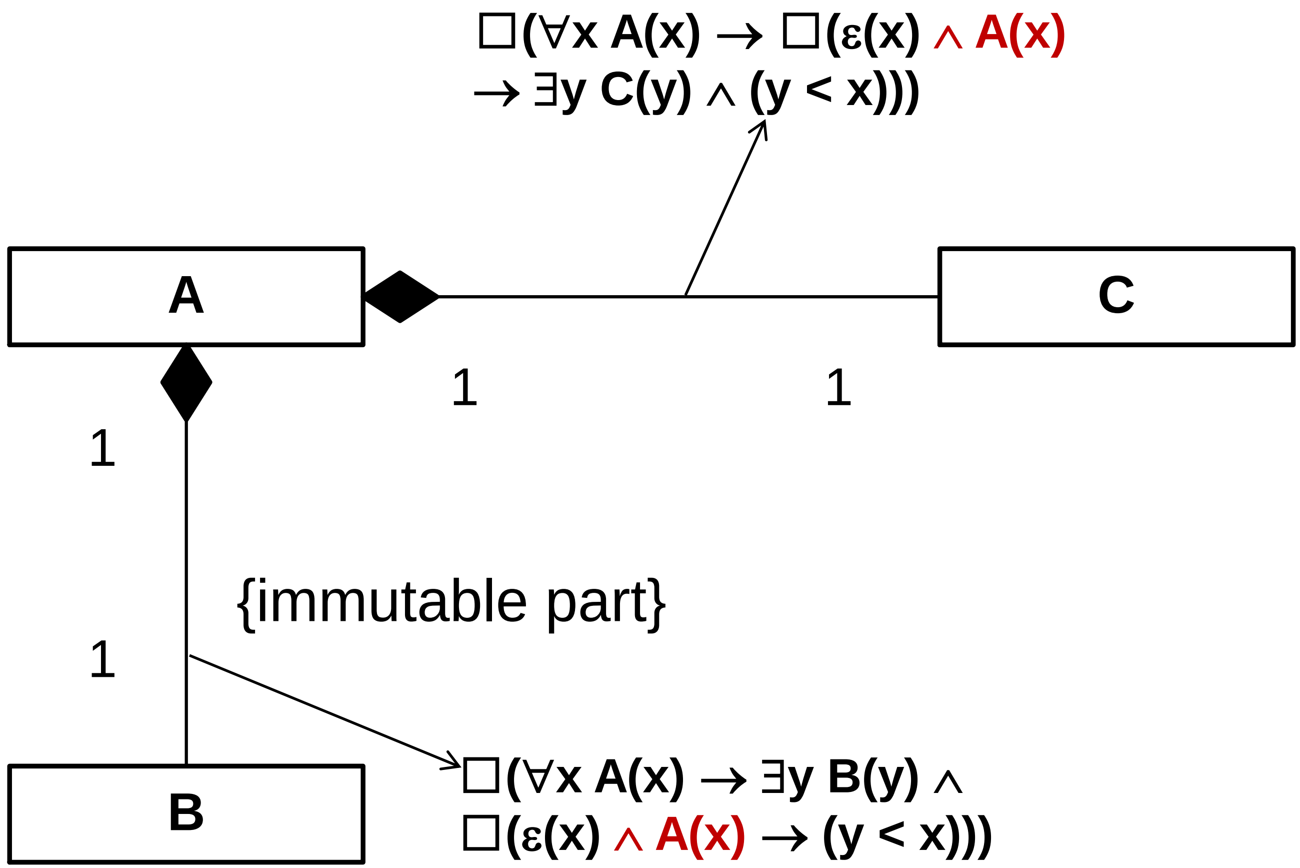
$\Box(\forall x \; \mathbf{Person}\,(x) \rightarrow \Box(\varepsilon(x) \wedge \mathbf{Person}(x) \rightarrow \exists! y \; \mathbf{Heart}(y) \wedge (y < x)))$

Immutable Part



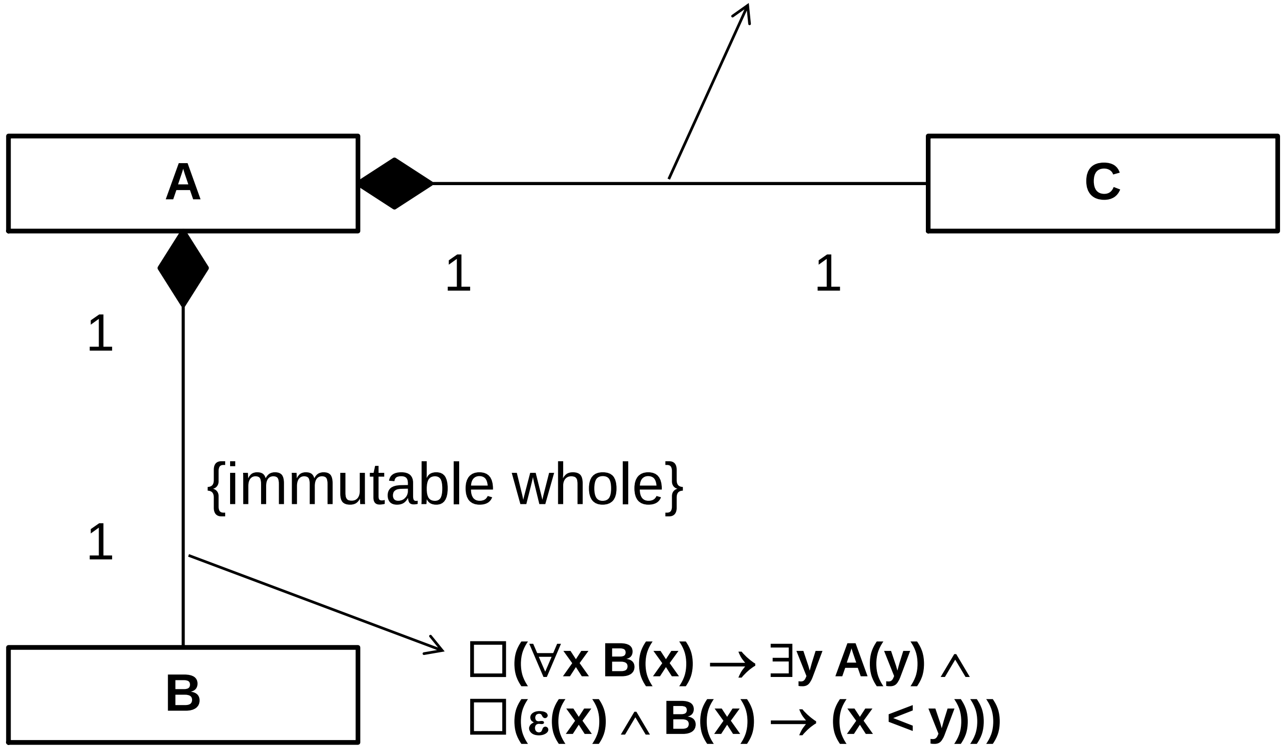
$$\Box(\forall x\ \mathrm{Boxer}(x) \rightarrow \exists y\ \mathrm{hand}(y) \land \Box(\varepsilon(x) \land \mathrm{Boxer}(x) \rightarrow (y < x)))$$


□(∀x A(x) → □(ε(x) ∧ A(x)
→ ∃y C(y) ∧ (y < x)))
A
C
1
1
1
{immutable part}
1
B
□(∀x A(x) → ∃y B(y) ∧
□(ε(x) ∧ A(x) → (y < x)))



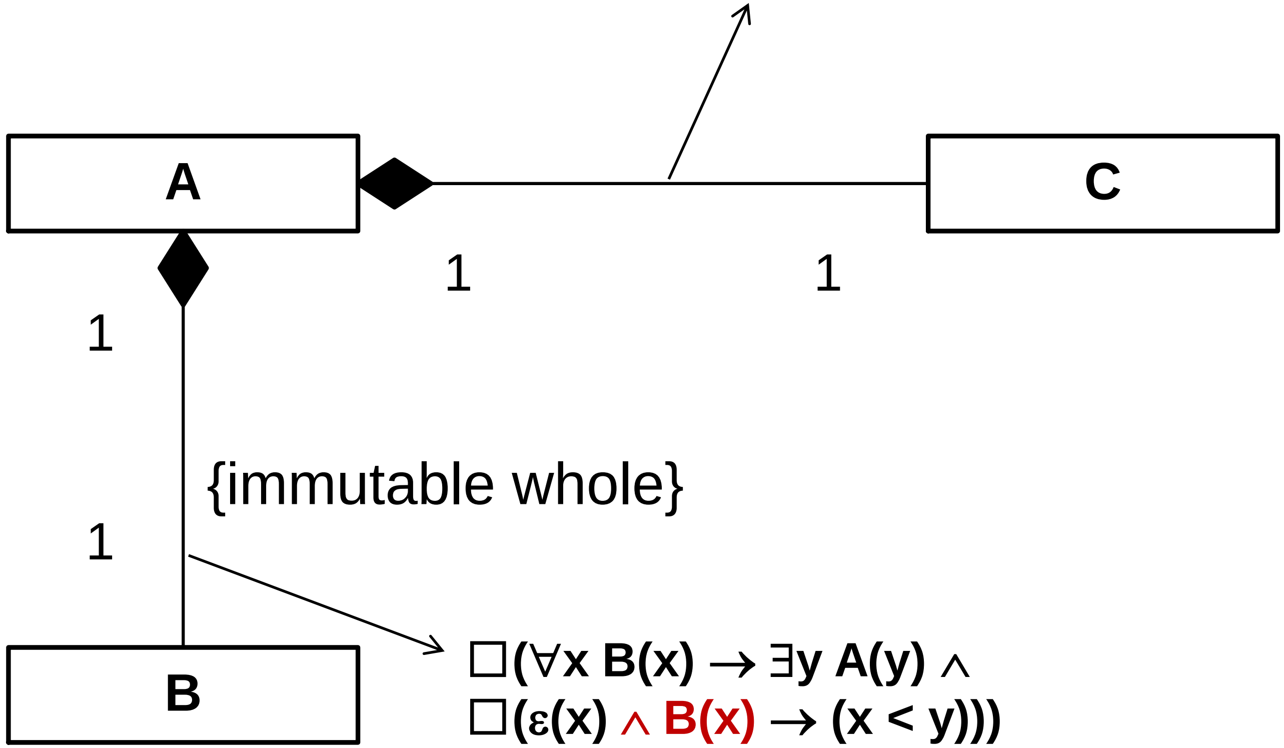
If A is a rigid type then B becomes an essential part of A and the formulae can be simplified

$$\Box(\forall x\, C(x) \rightarrow \Box(\varepsilon(x) \wedge C(x) \rightarrow \exists y\, A(y) \wedge (x < y)))$$



$$\Box(\forall x\, B(x) \rightarrow \exists y\, A(y) \wedge \Box(\varepsilon(x) \wedge B(x) \rightarrow (x < y)))$$

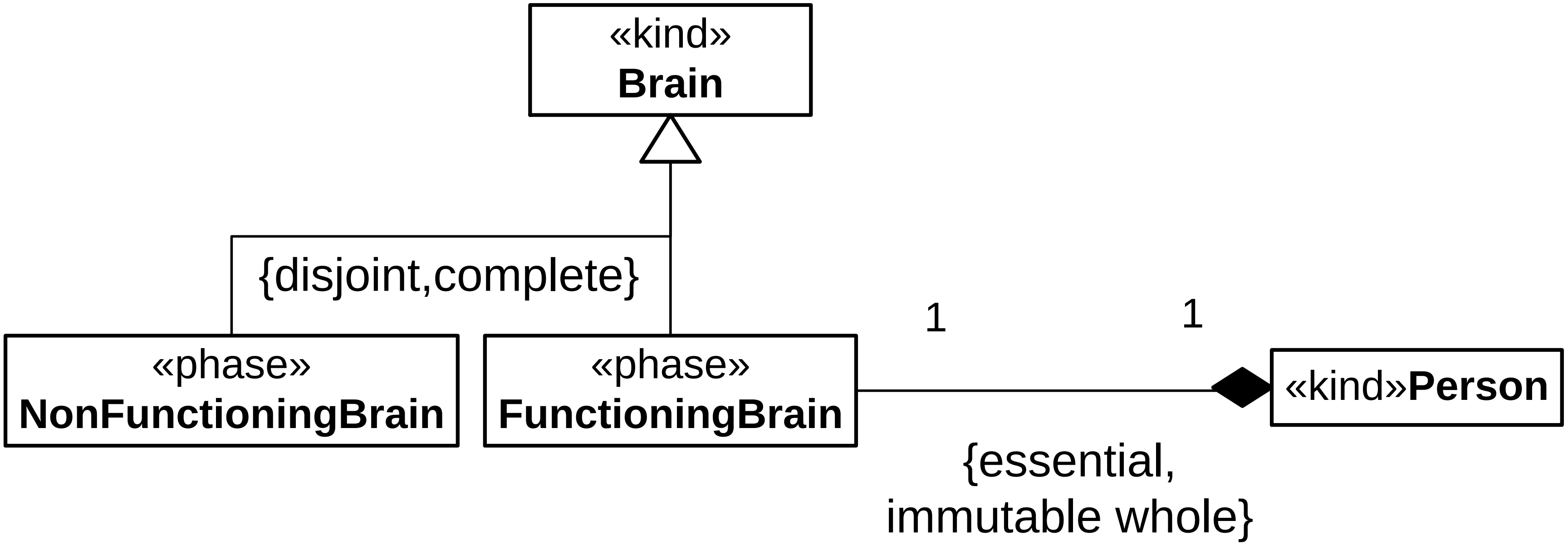
$\Box(\forall x\ C(x) \rightarrow \Box(\varepsilon(x) \wedge C(x) \rightarrow \exists y\ A(y) \wedge (x < y)))$



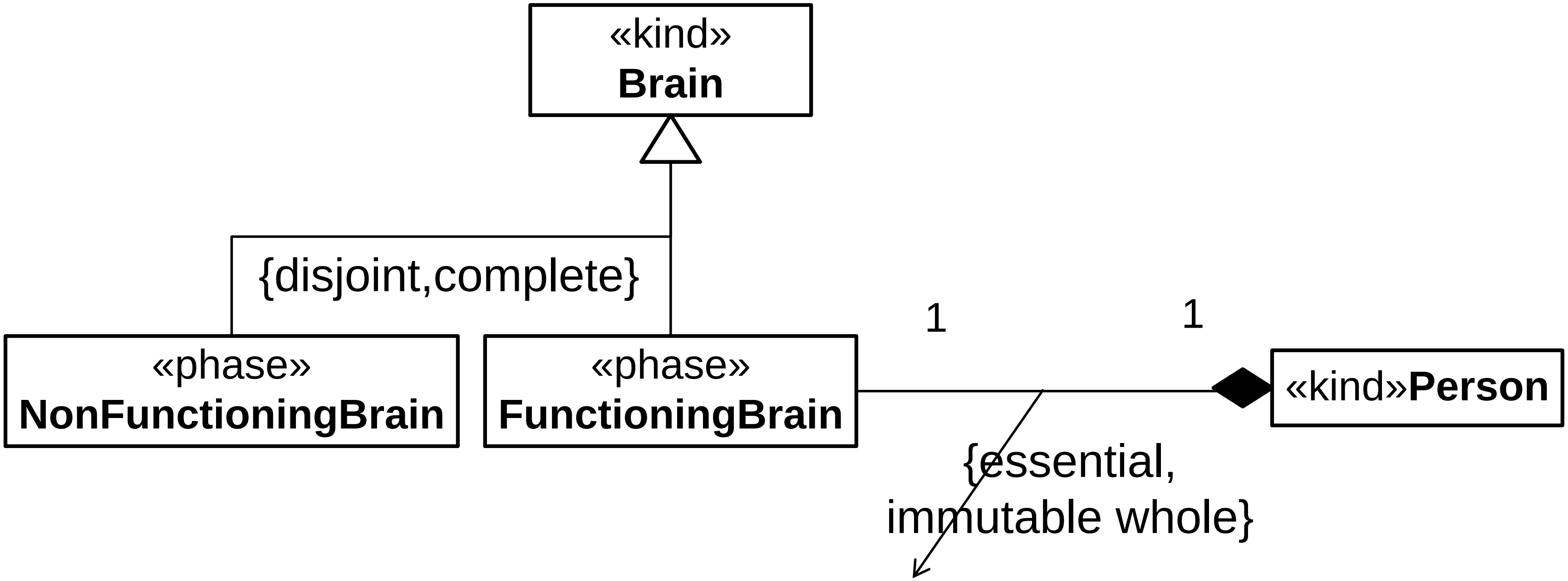
$\Box(\forall x\ B(x) \rightarrow \exists y\ A(y) \wedge \Box(\varepsilon(x) \wedge B(x) \rightarrow (x < y)))$

If A is a rigid type then B becomes an inseparable part of A and the formulae can be simplified

Example of Immutable Whole



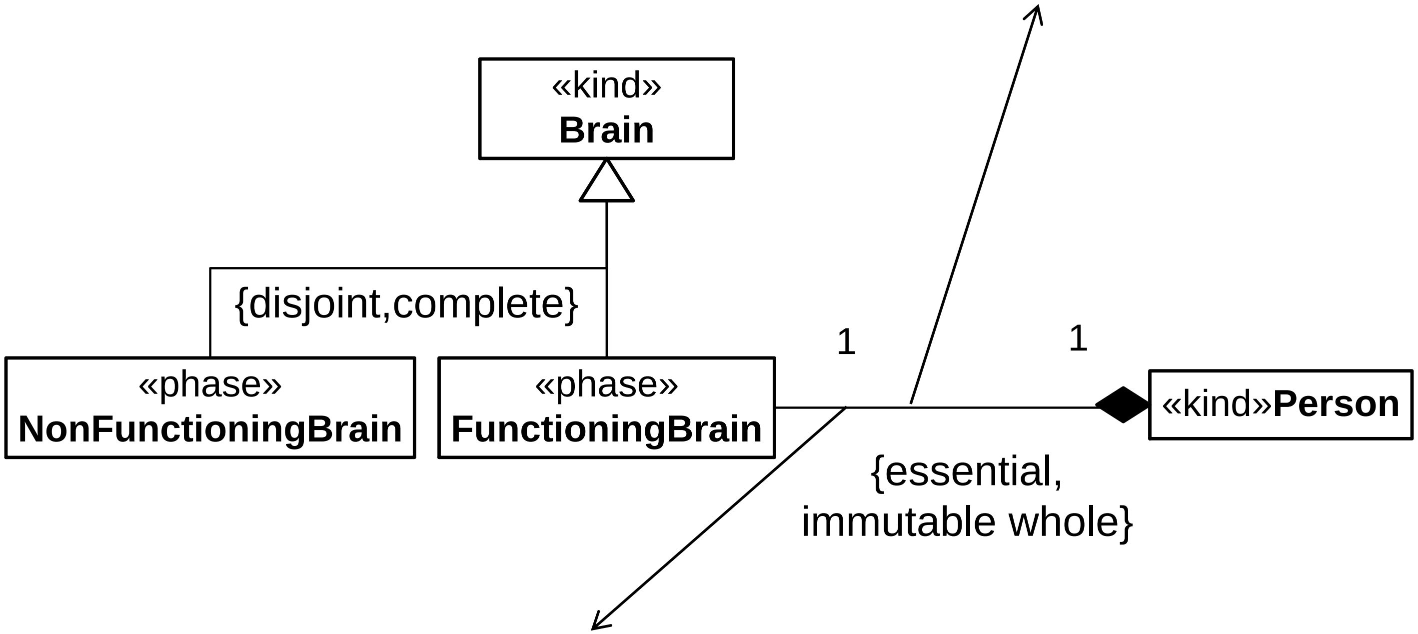
Example of Immutable Whole



$$\Box(\forall x\ \mathbf{FunctioningBrain}(x) \rightarrow \exists! y\ \mathbf{Person}(y) \wedge \Box(\varepsilon(x) \wedge \mathbf{FunctioningBrain}(x) \rightarrow (x < y)))$$

Example of Immutable Whole

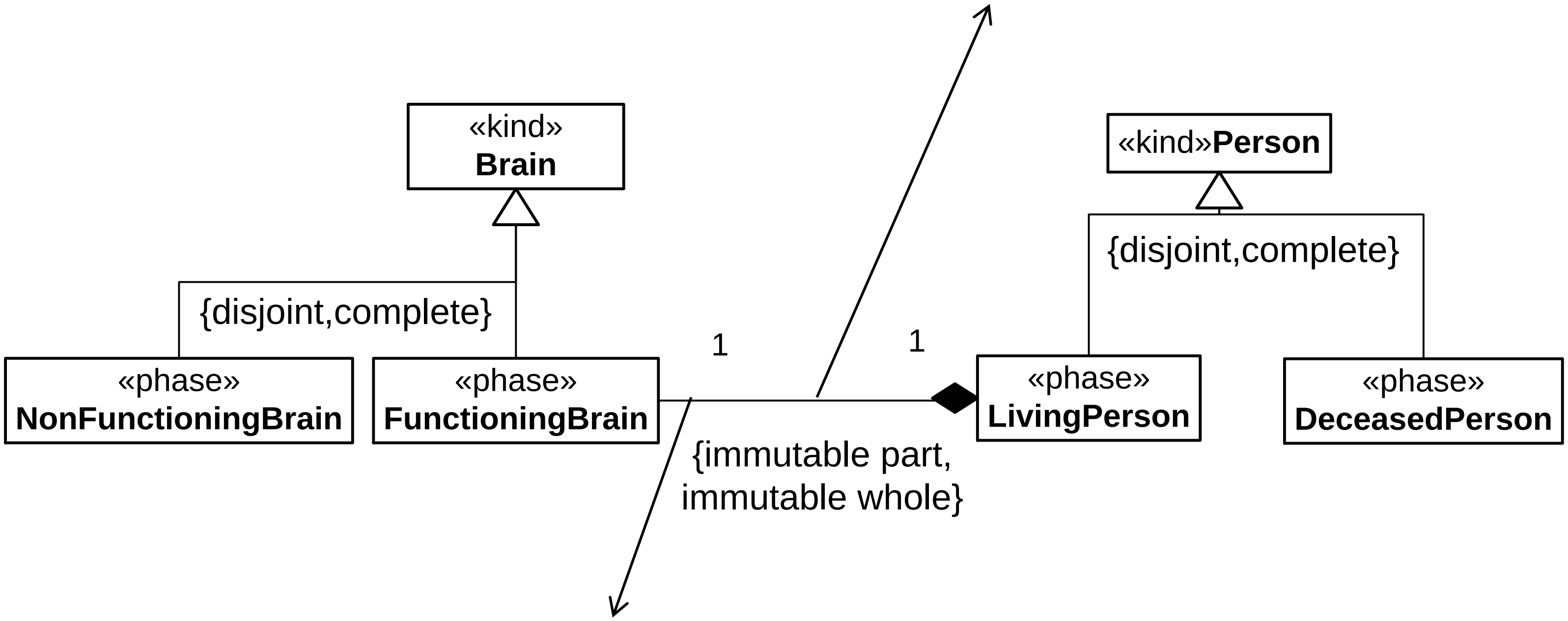
$$\Box(\forall x\ \mathrm{Person}(x) \rightarrow \exists! y\ \mathrm{FunctioningBrain}(y)$$
$$\Box(\varepsilon(x) \rightarrow \mathrm{FunctioningBrain}(y) \wedge (y < x)))$$



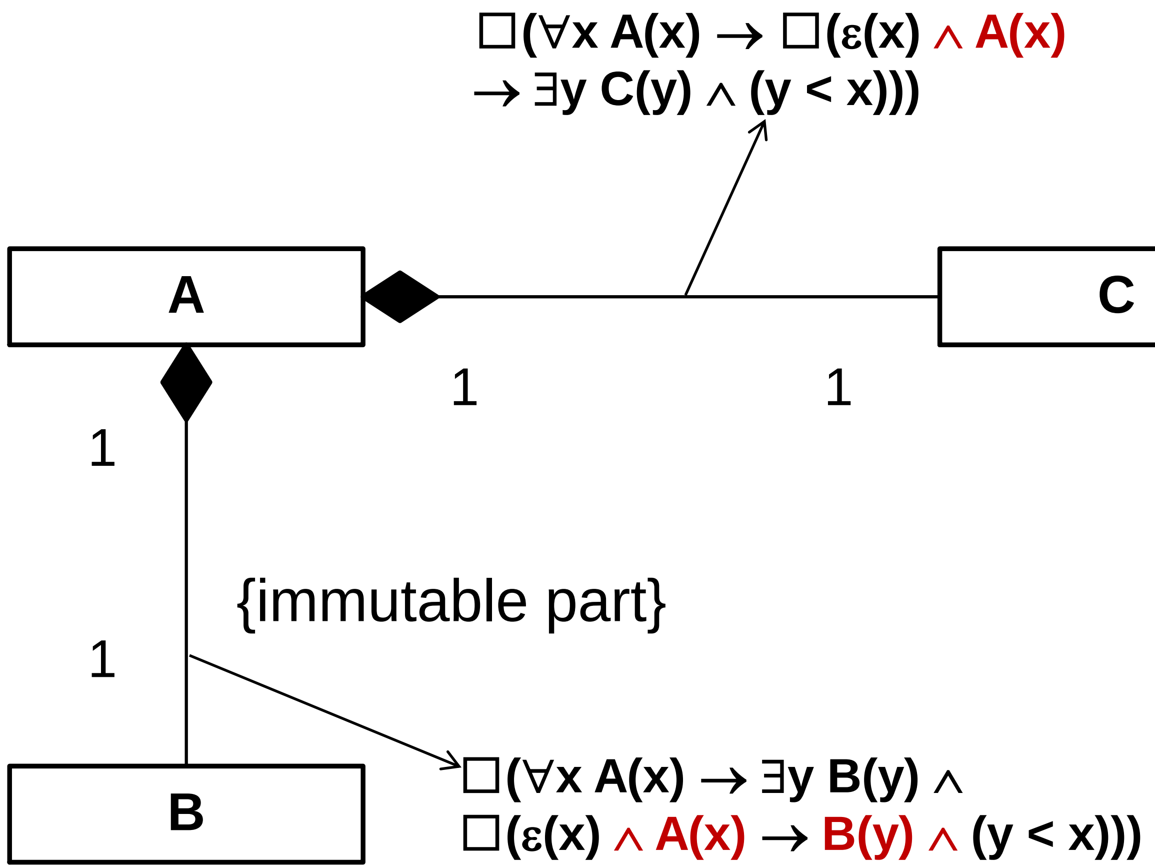
$$\Box(\forall x\ \mathrm{FunctioningBrain}(x) \rightarrow \exists! y\ \mathrm{Person}(y) \wedge \Box(\varepsilon(x)$$
$$\wedge\ \mathrm{FunctioningBrain}(x) \rightarrow (x < y)))$$

Immutable Whole and Immutable Part

$\Box(\forall x\ LivingPerson\ (x) \rightarrow \exists! y\ FunctioningBrain(y)$
$\Box(\varepsilon(x) \wedge LivingPerson(y) \rightarrow FunctioningBraing(x) \wedge (y < x)))$

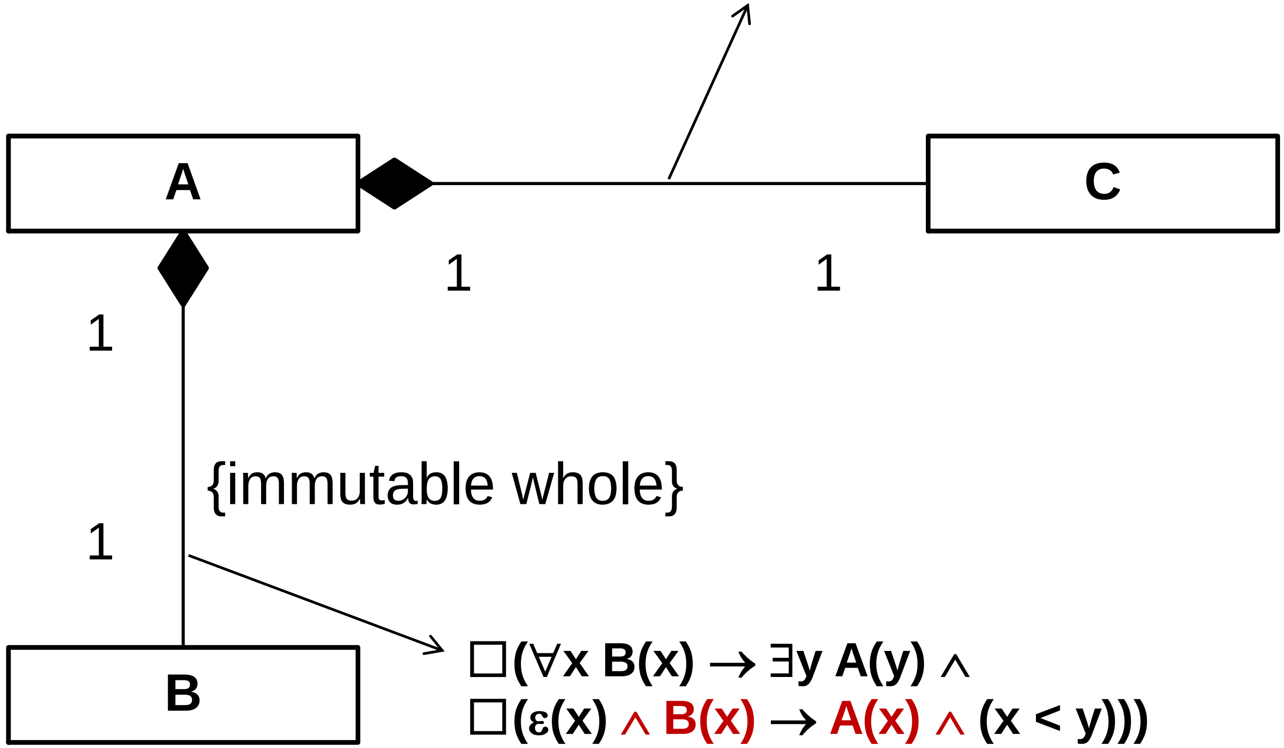


$\Box(\forall x\ FunctioningBrain(x) \rightarrow \exists! y\ LivingPerson(y) \wedge \Box(\varepsilon(x)$
$\wedge\ FunctioningBrain(x) \rightarrow LivingPerson(y) \wedge (x < y)))$



If A and B are rigid types then the predicates in red can be omitted

$$\Box(\forall x\, C(x) \rightarrow \Box(\varepsilon(x) \wedge C(x) \rightarrow \exists y\, A(y) \wedge (x < y)))$$

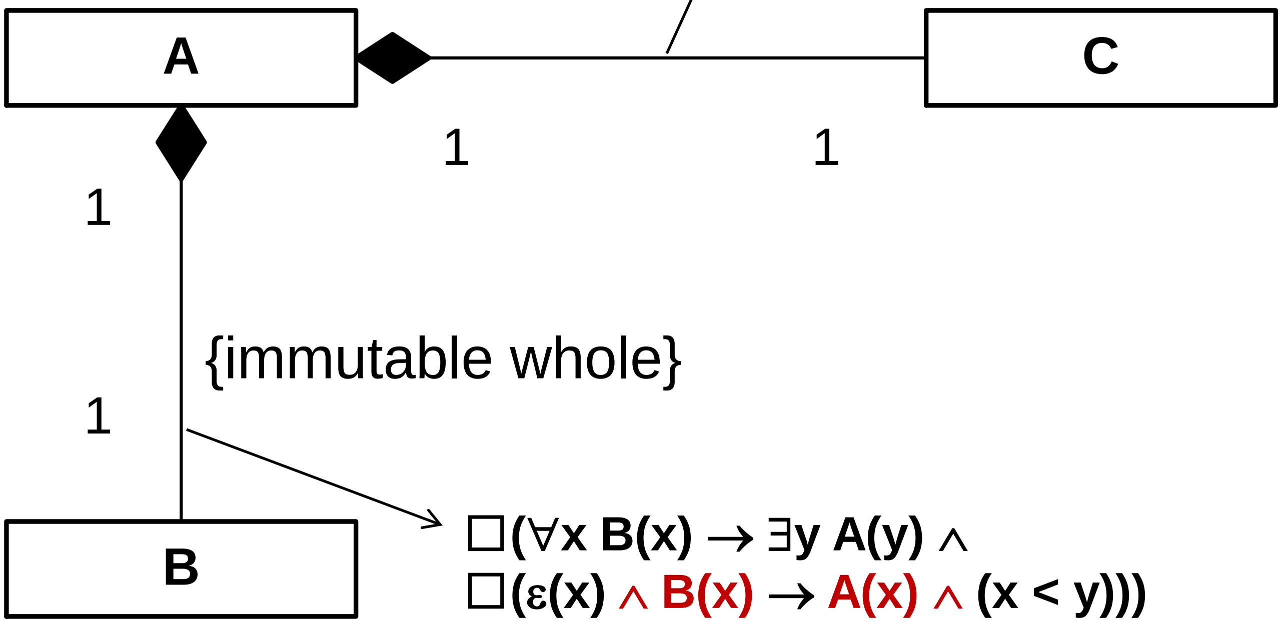


$$\Box(\forall x\, B(x) \rightarrow \exists y\, A(y) \wedge \Box(\varepsilon(x) \wedge B(x) \rightarrow A(x) \wedge (x < y)))$$

If A and B are rigid types then the predicates in red can be omitted

For the case
of mandatory parts
and mandatory wholes,
these remain the same

$$\Box(\forall x\, C(x) \rightarrow \Box(\varepsilon(x) \wedge C(x) \rightarrow \exists y\, A(y) \wedge (x < y)))$$

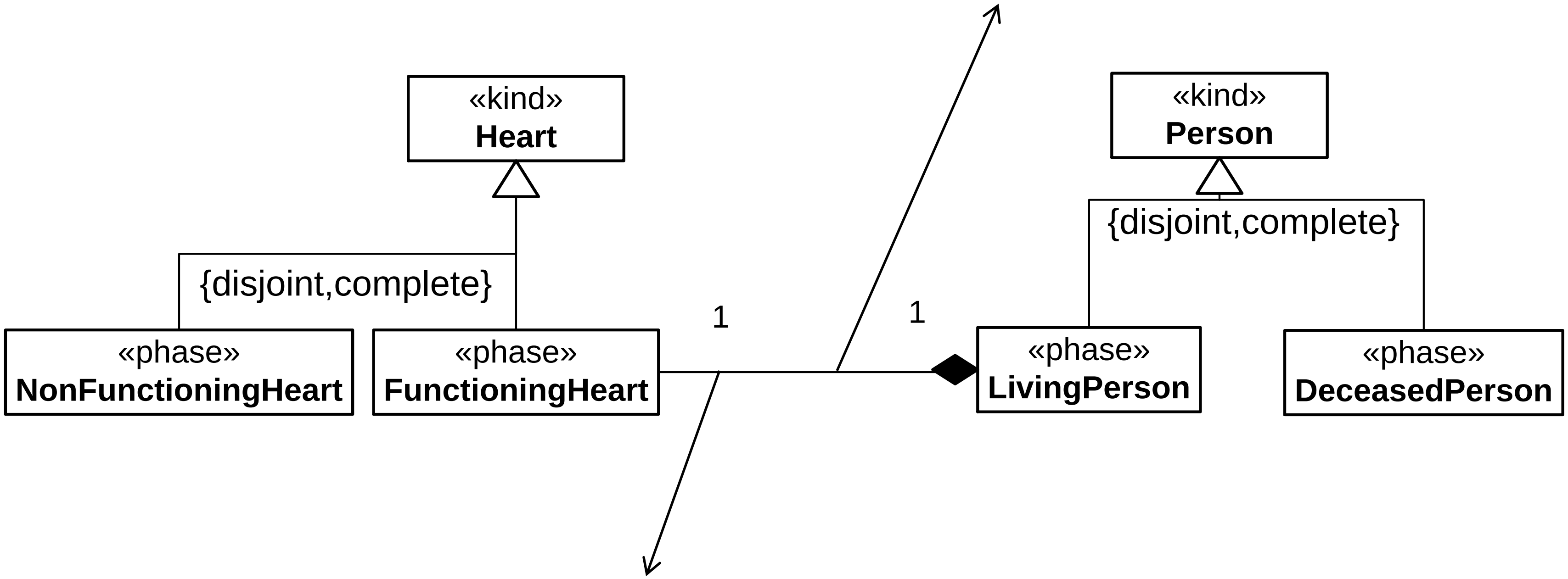


$$\Box(\forall x\, B(x) \rightarrow \exists y\, A(y) \wedge \Box(\varepsilon(x) \wedge B(x) \rightarrow A(x) \wedge (x < y)))$$

If A and B are rigid types then the predicates in red can be omitted

Mandatory Part and Mandatory Whole

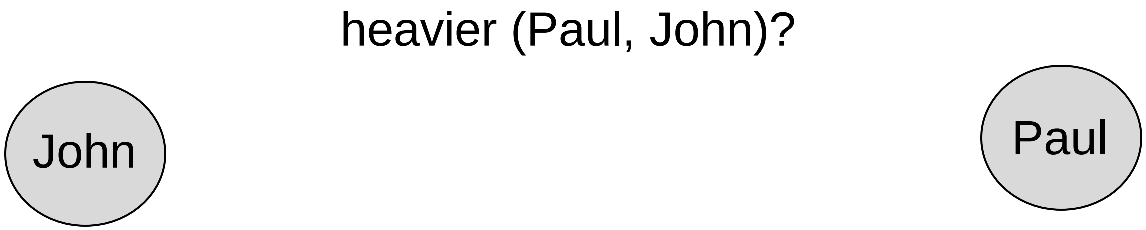
$$\Box(\forall x\ LivingPerson(x) \rightarrow \Box(\varepsilon(x) \wedge LivingPerson(x) \rightarrow \exists y\ FunctioningHeart(y) \wedge (y < x)))$$



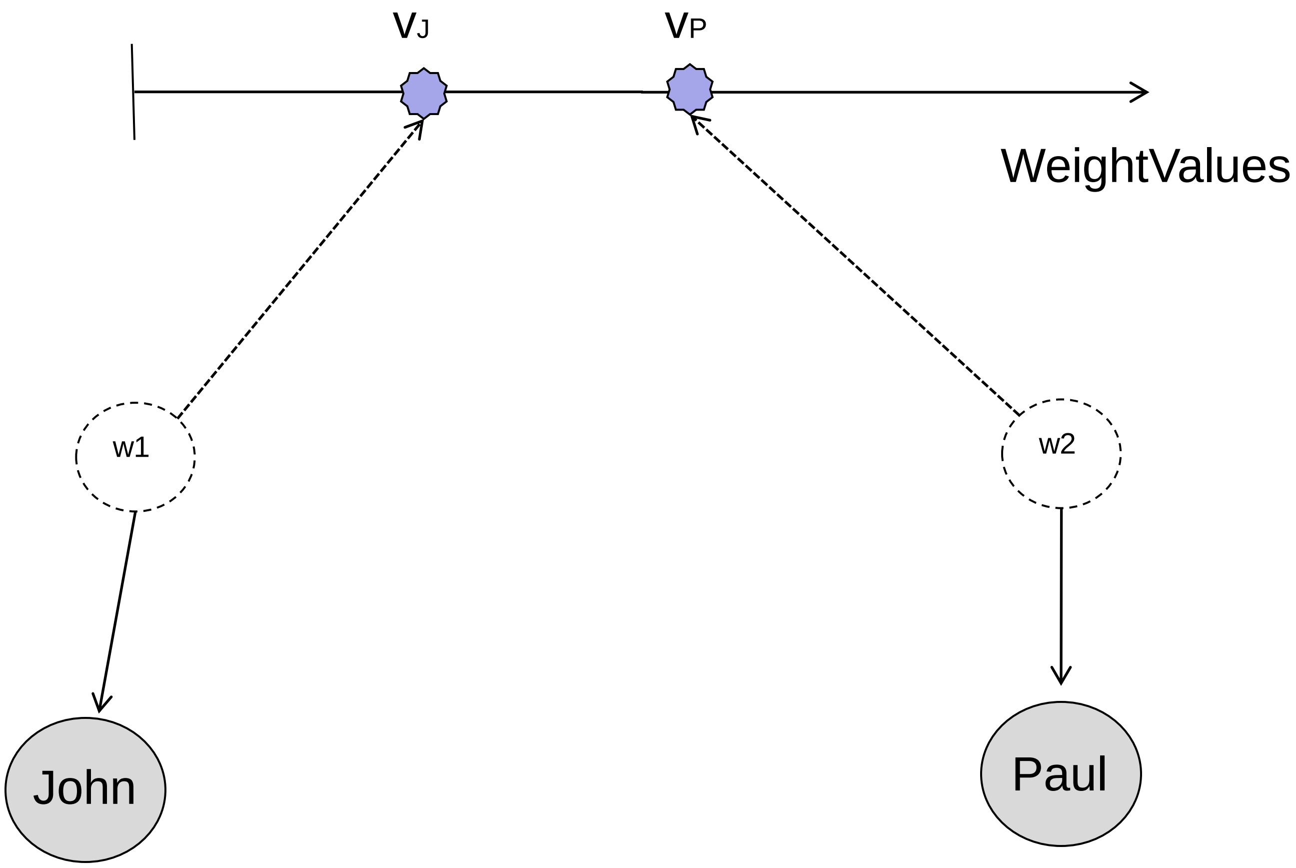
$$\Box(\forall x\ FunctioningHeart(x) \rightarrow \Box(\varepsilon(x) \wedge FunctioningHeart(x) \rightarrow \exists y\ LivingPerson(y) \wedge (x < y)))$$

RELATIONS

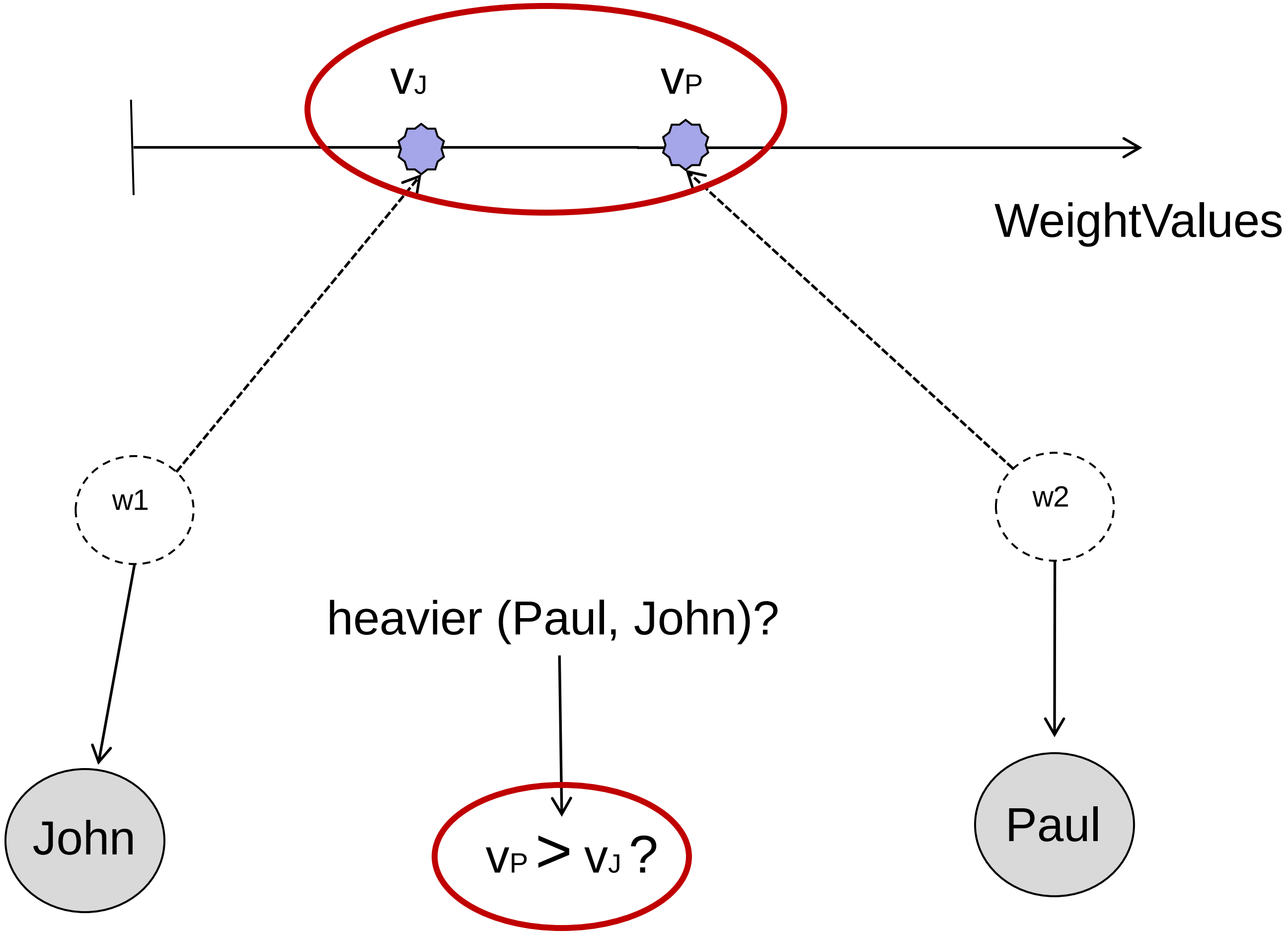
Formal Relations

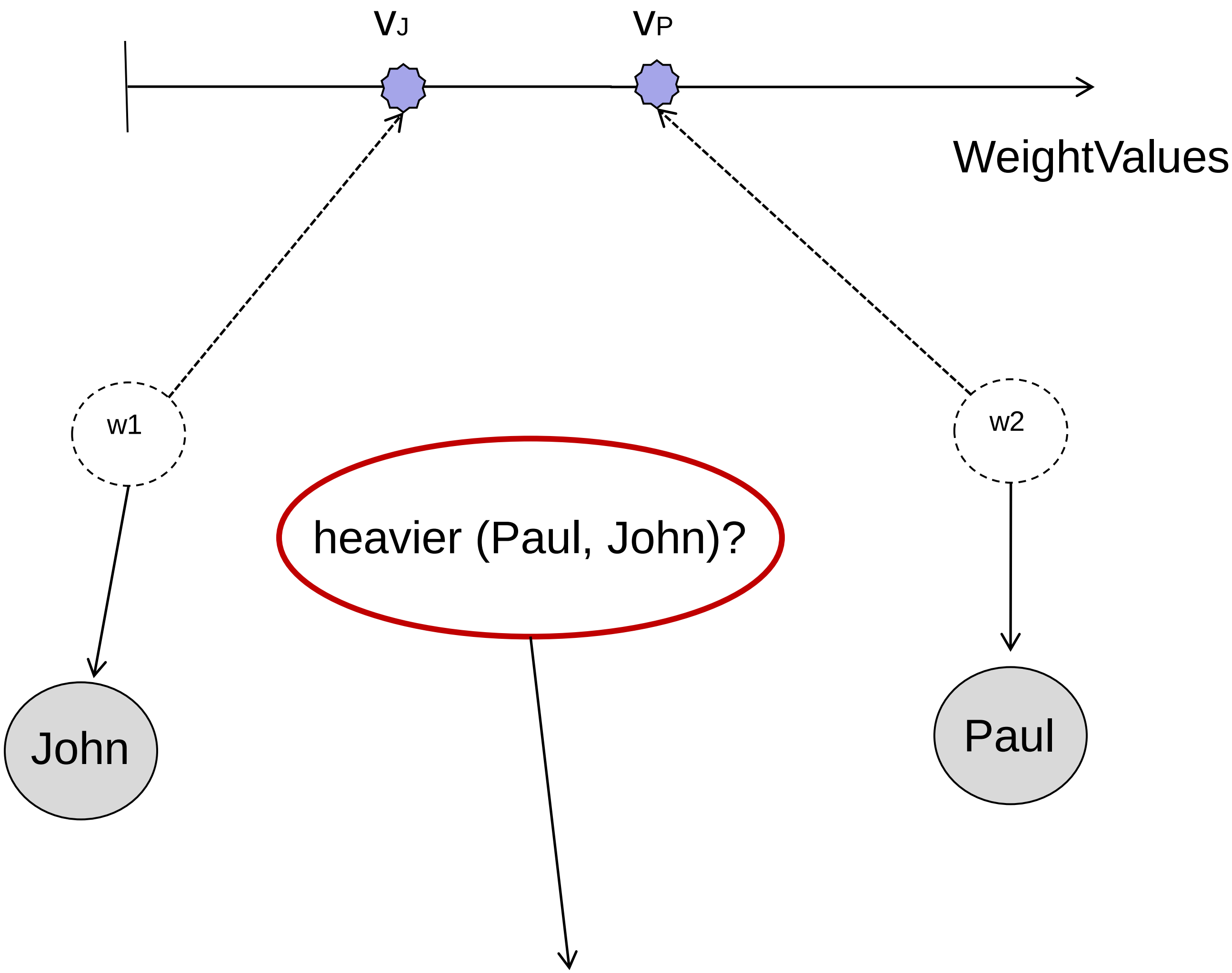


Formal Relations

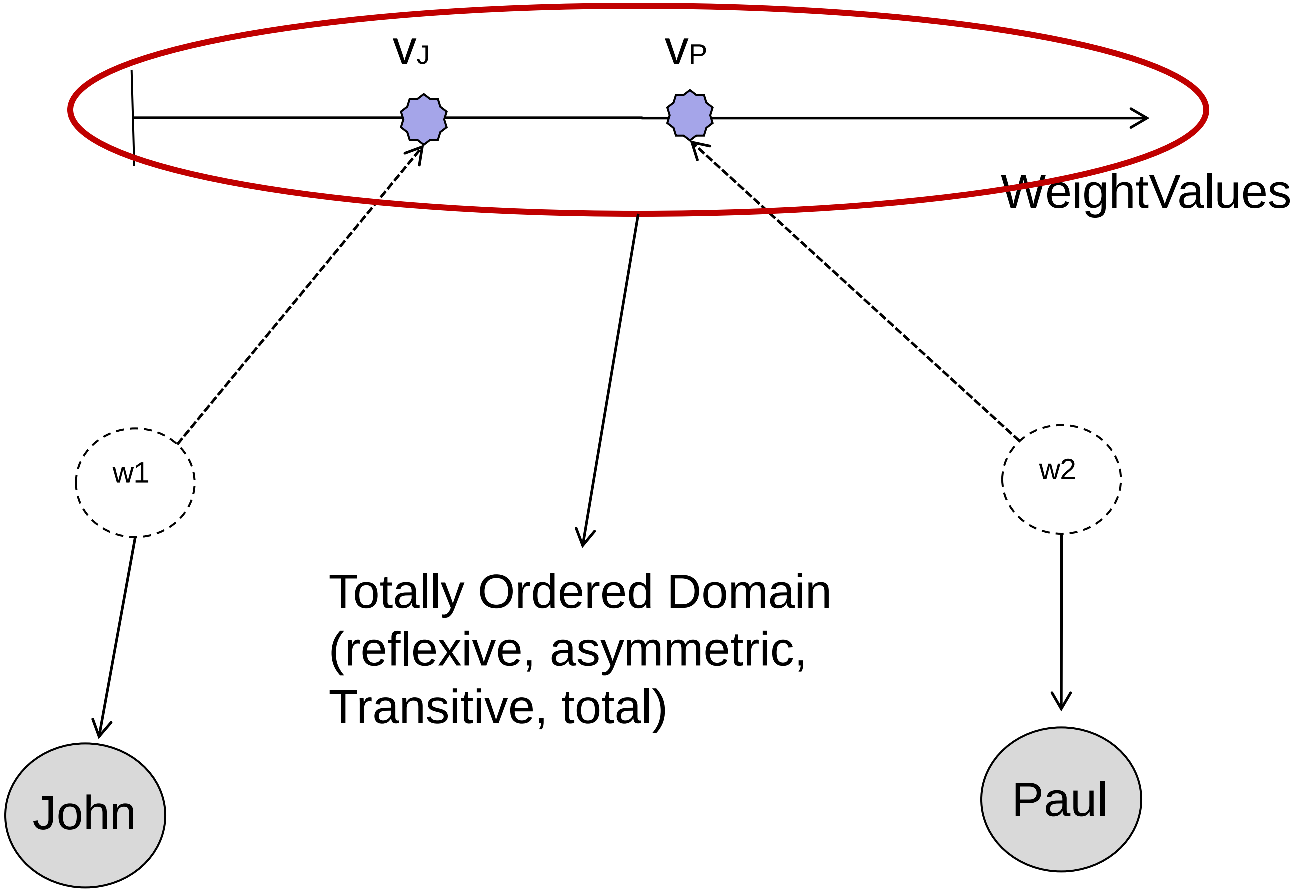


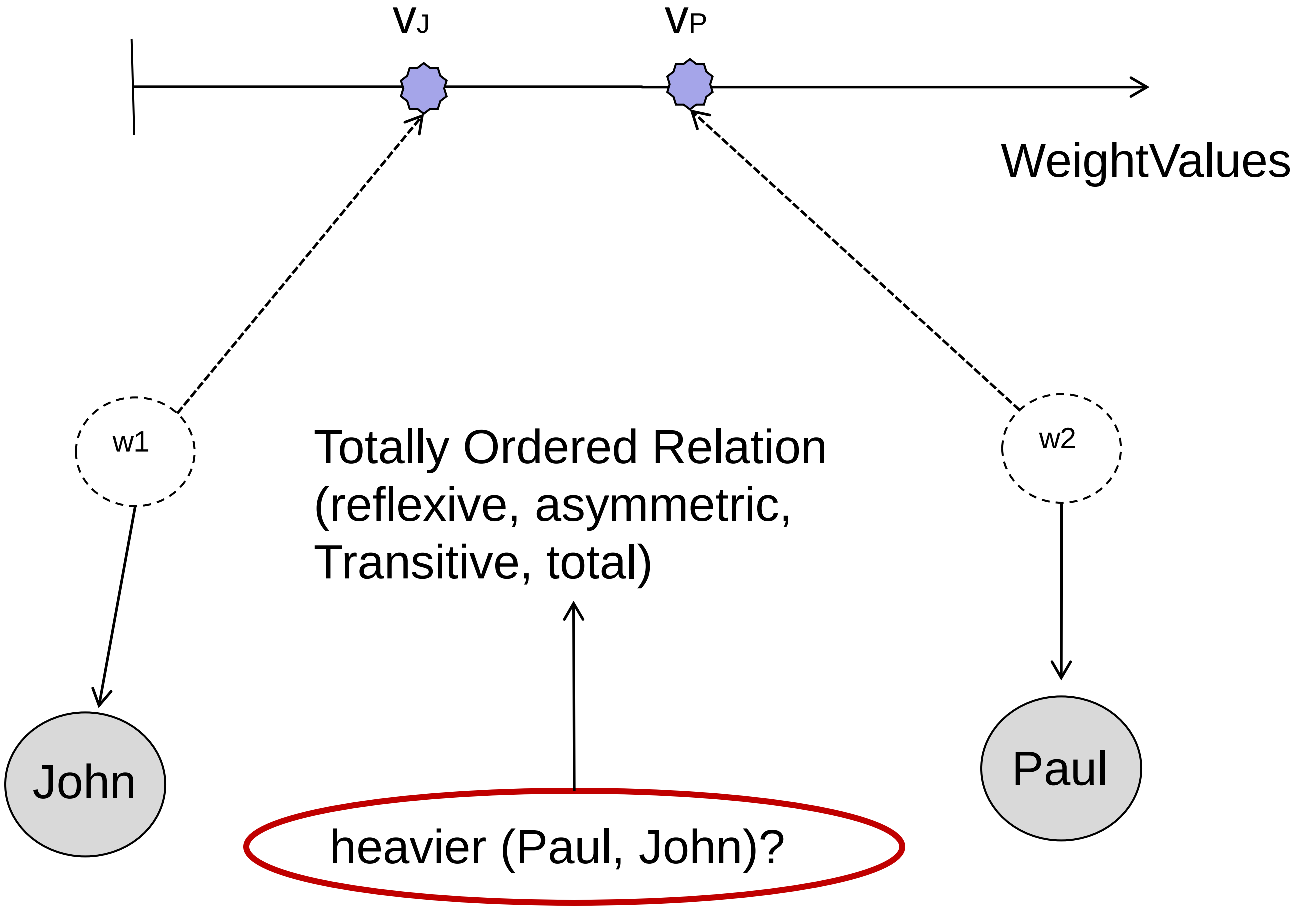
Formal Relations



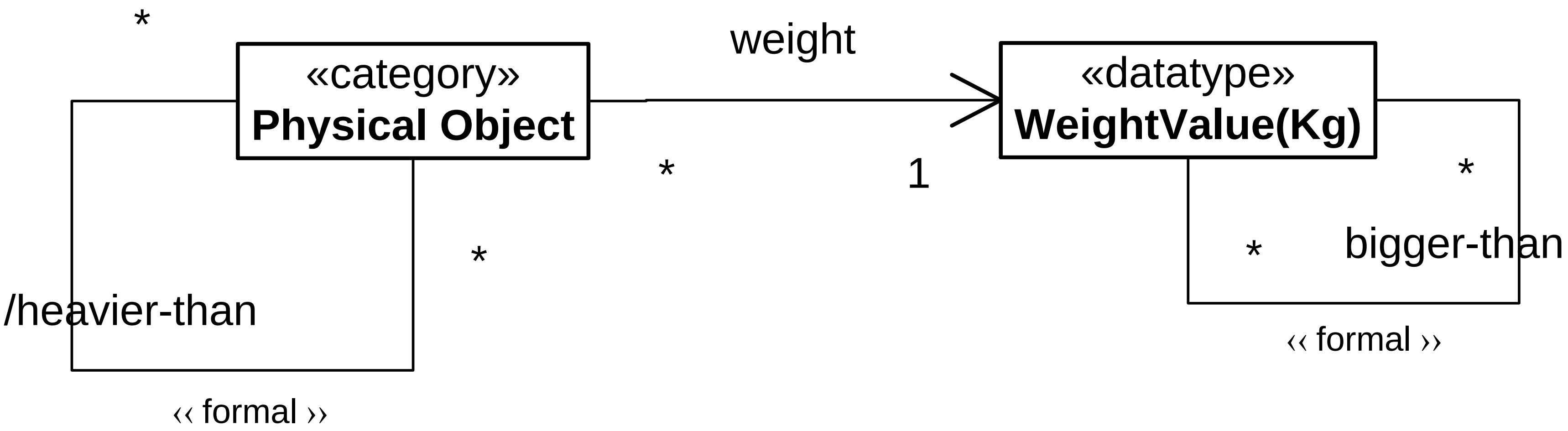


Notice that the meta-properties of this relation are derived from the meta-properties of the underlying quality structure


VJ
VP
WeightValues
w1
w2
Totally Ordered Domain
(reflexive, asymmetric,
Transitive, total)
John
Paul

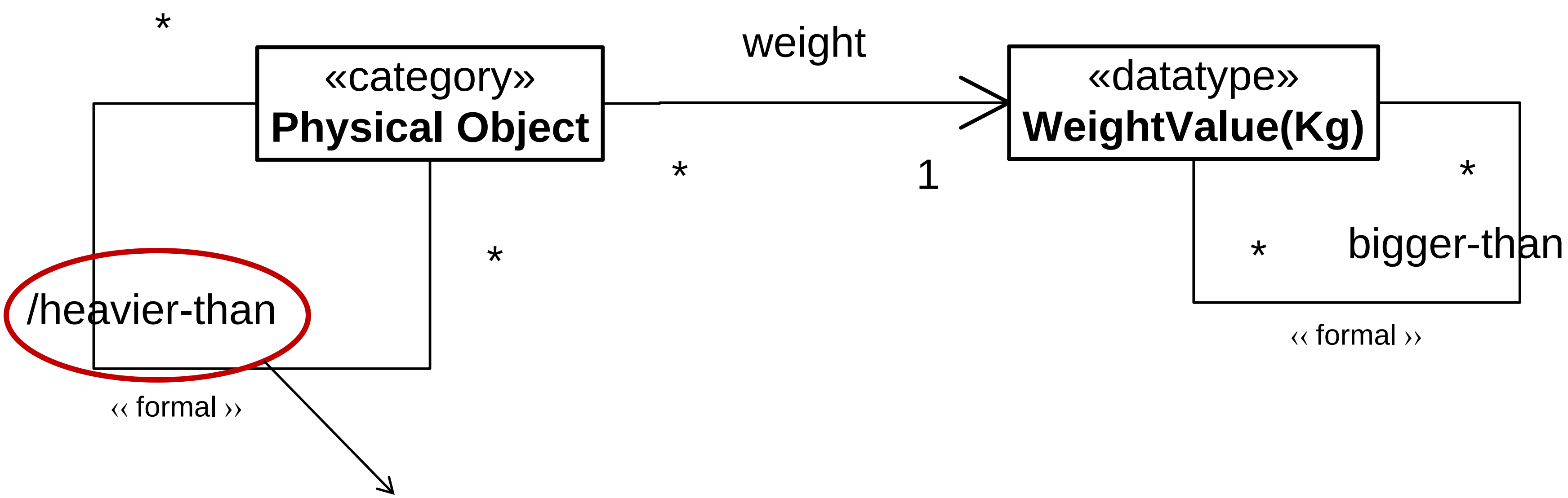

VJ
VP
WeightValues
w1
w2
Totally Ordered Relation
(reflexive, asymmetric,
Transitive, total)
John
Paul
heavier (Paul, John)?

Domain Formal Relations



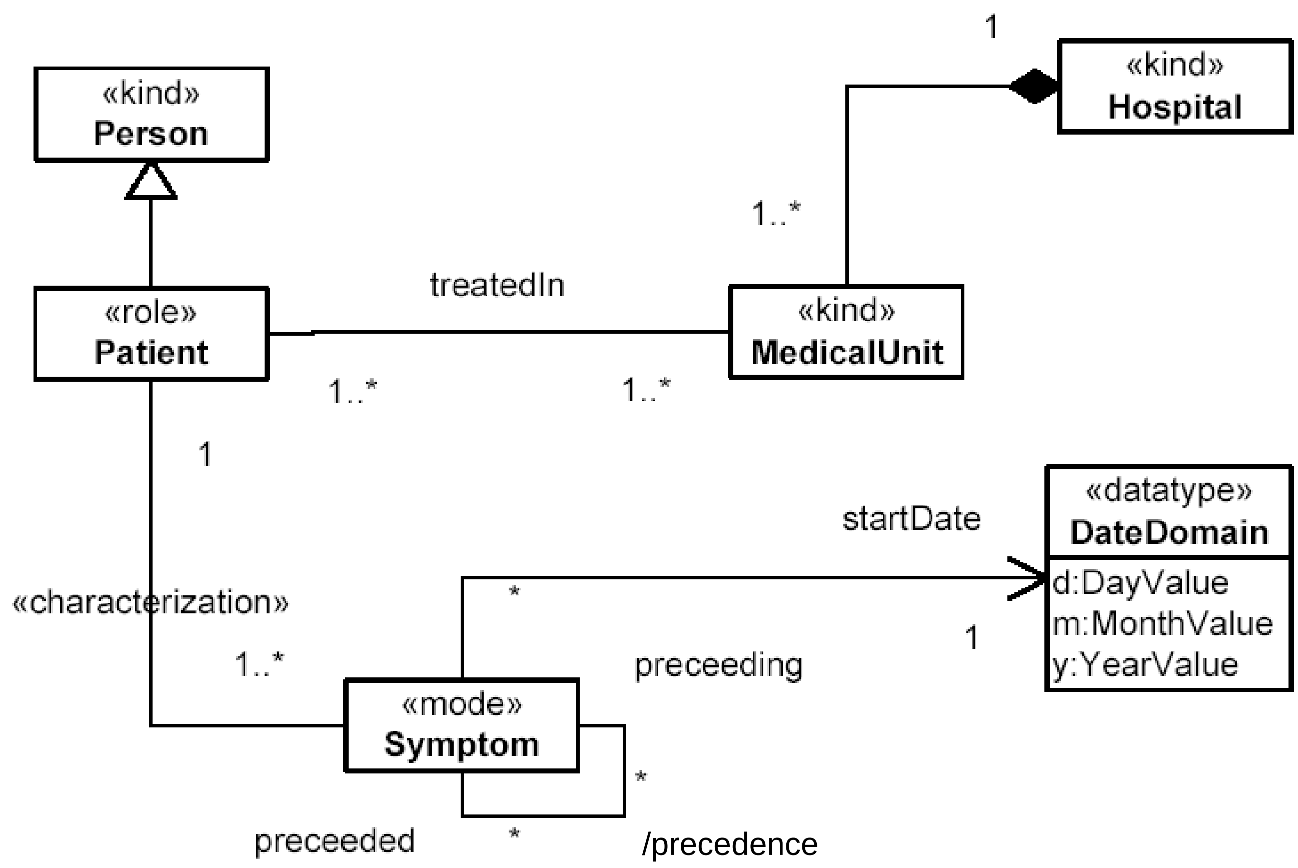
Domain Formal Relations are always derived relations, i.e., relations which can be dynamically computed (e.g., via queries) from other elements in the model

Domain Formal Relations

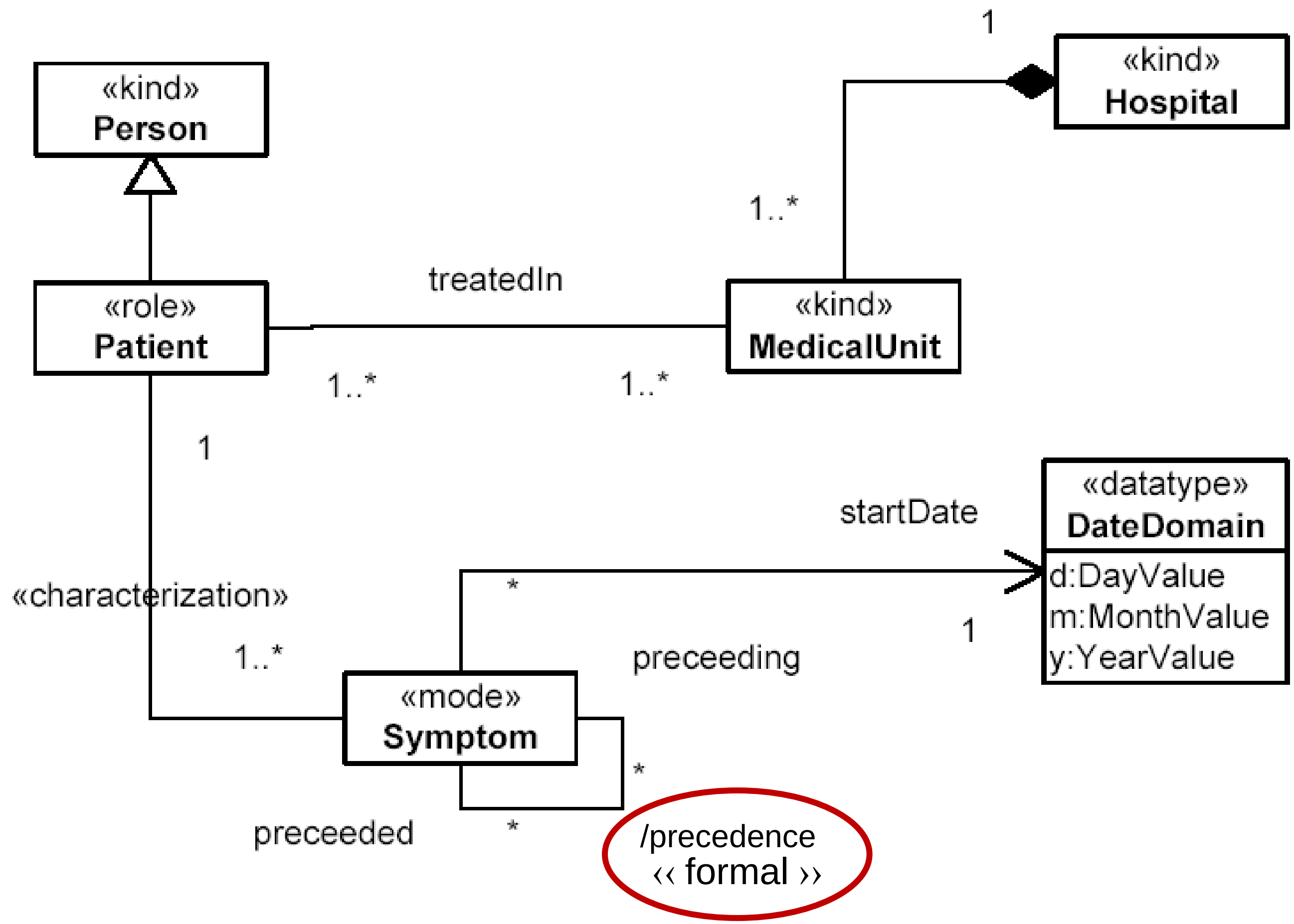


heavier-than(x,y) =$_{def}$ bigger-than(weight(x),weight(y))

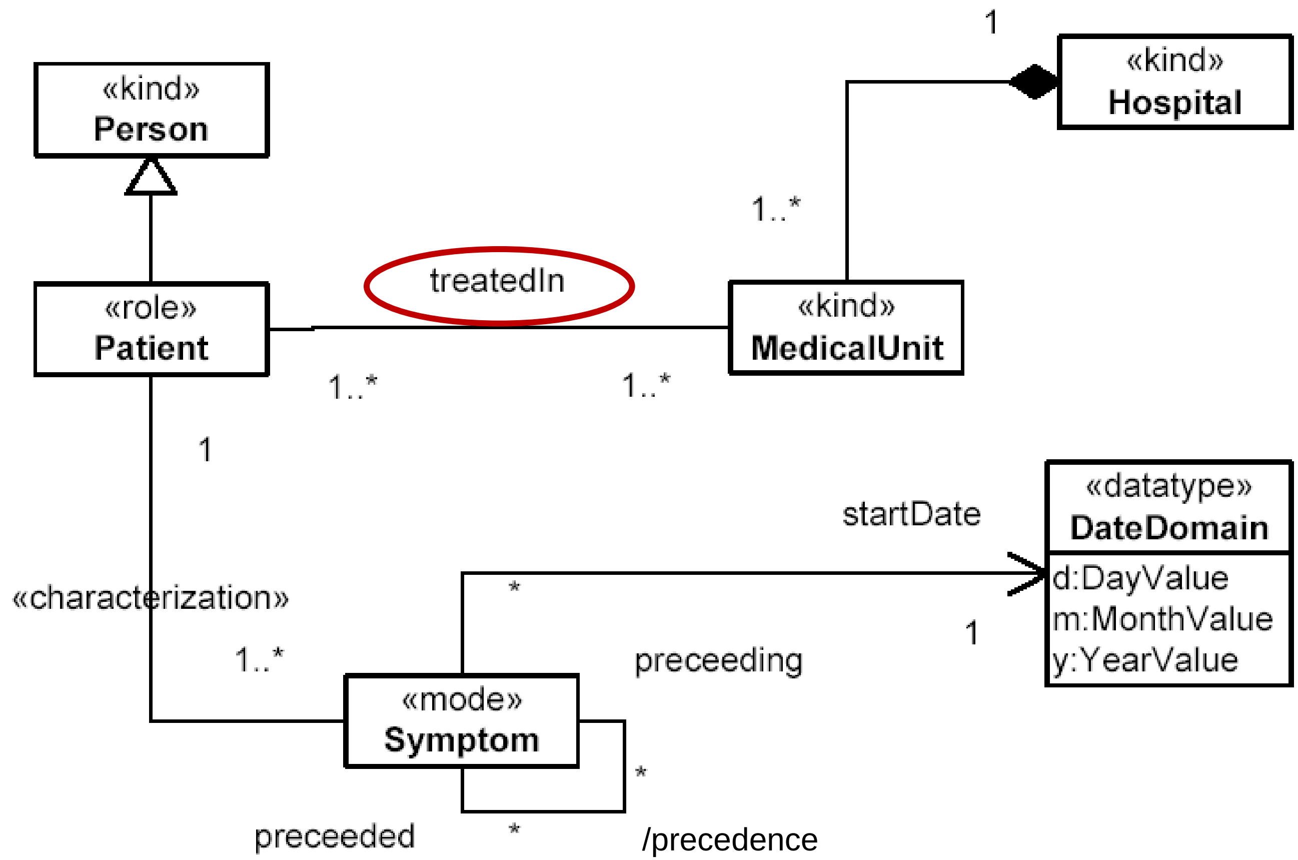
Formal and Material Relations



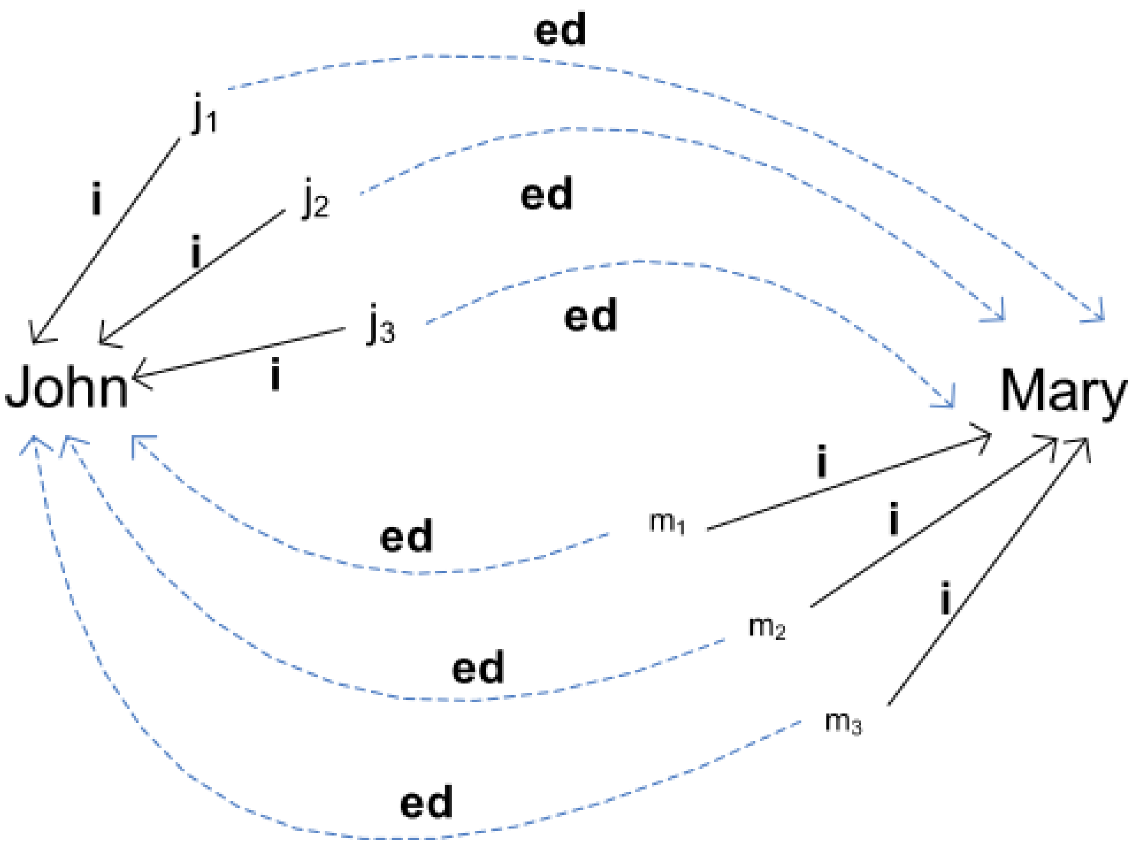
Formal and Material Relations



Formal and Material Relations

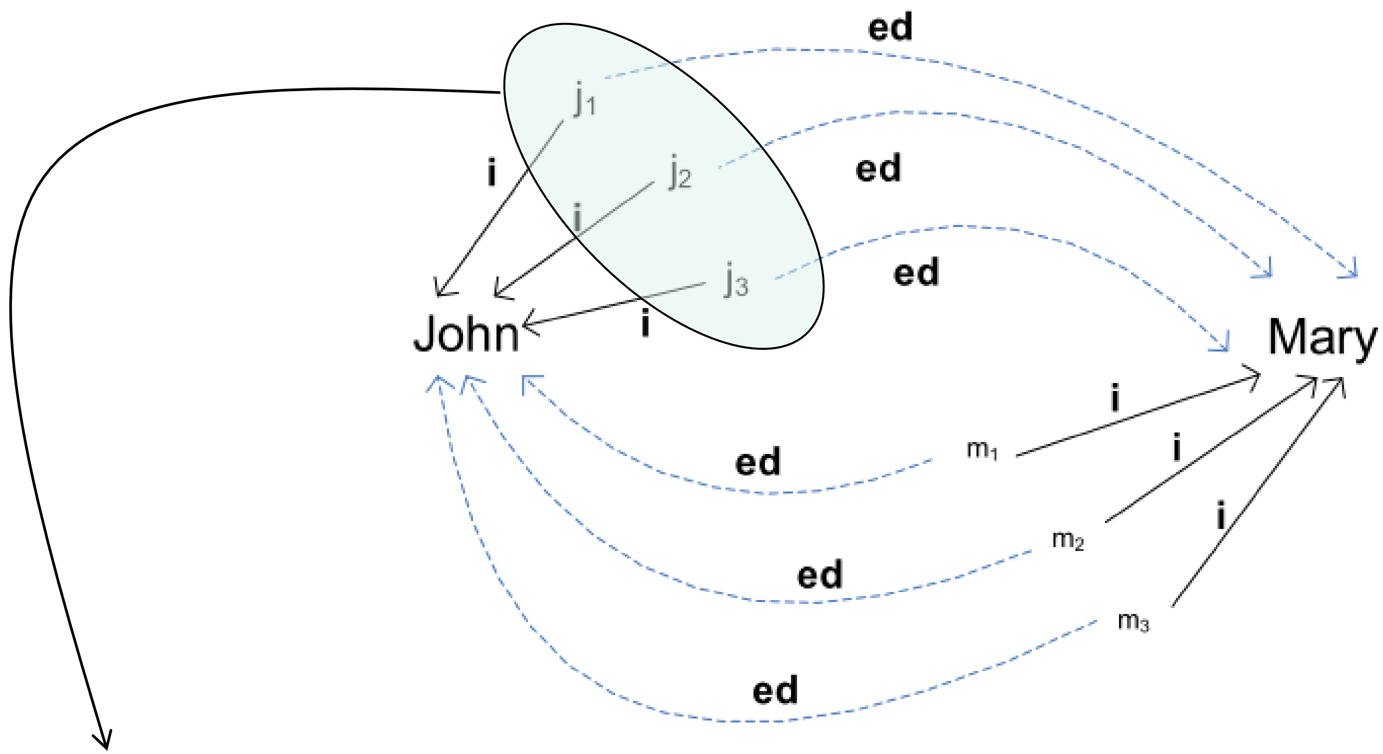


Externally Dependent Aspect



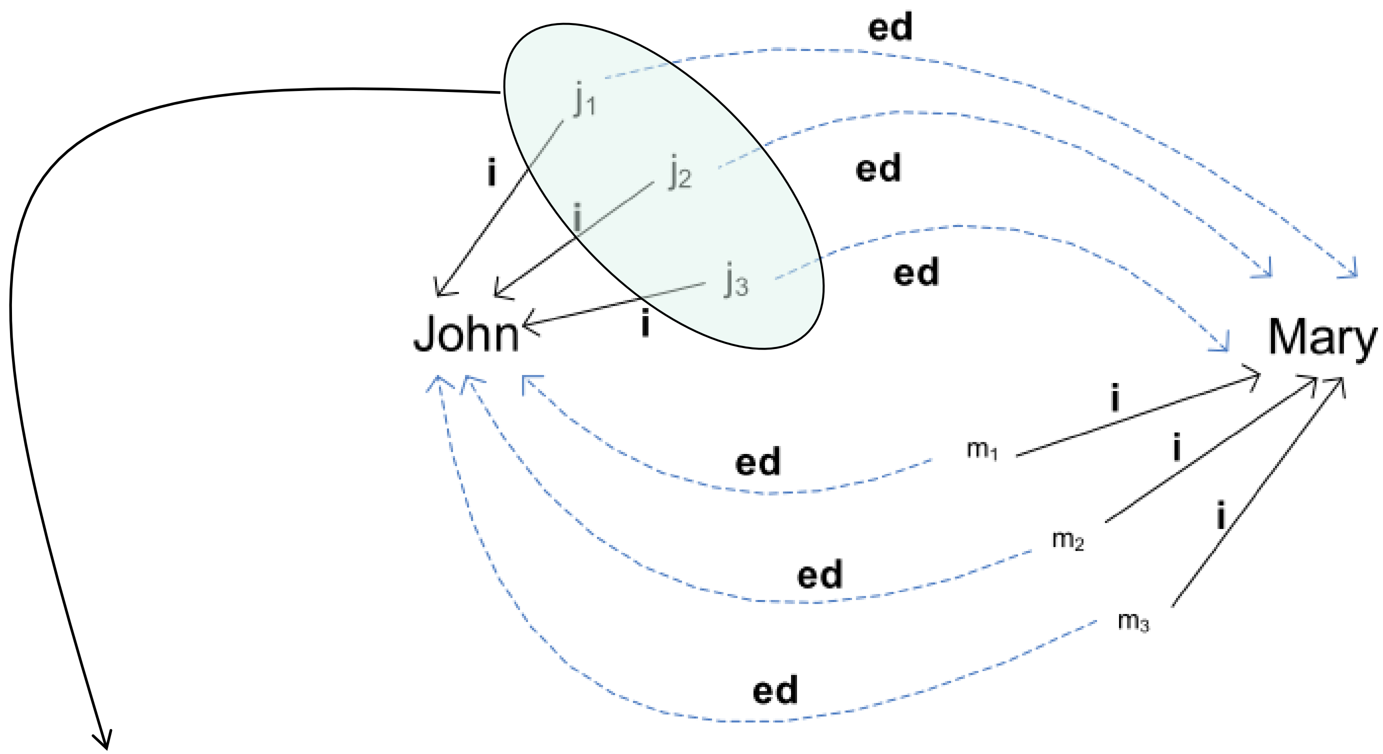
Suppose John marries Mary

Externally Dependent Aspect



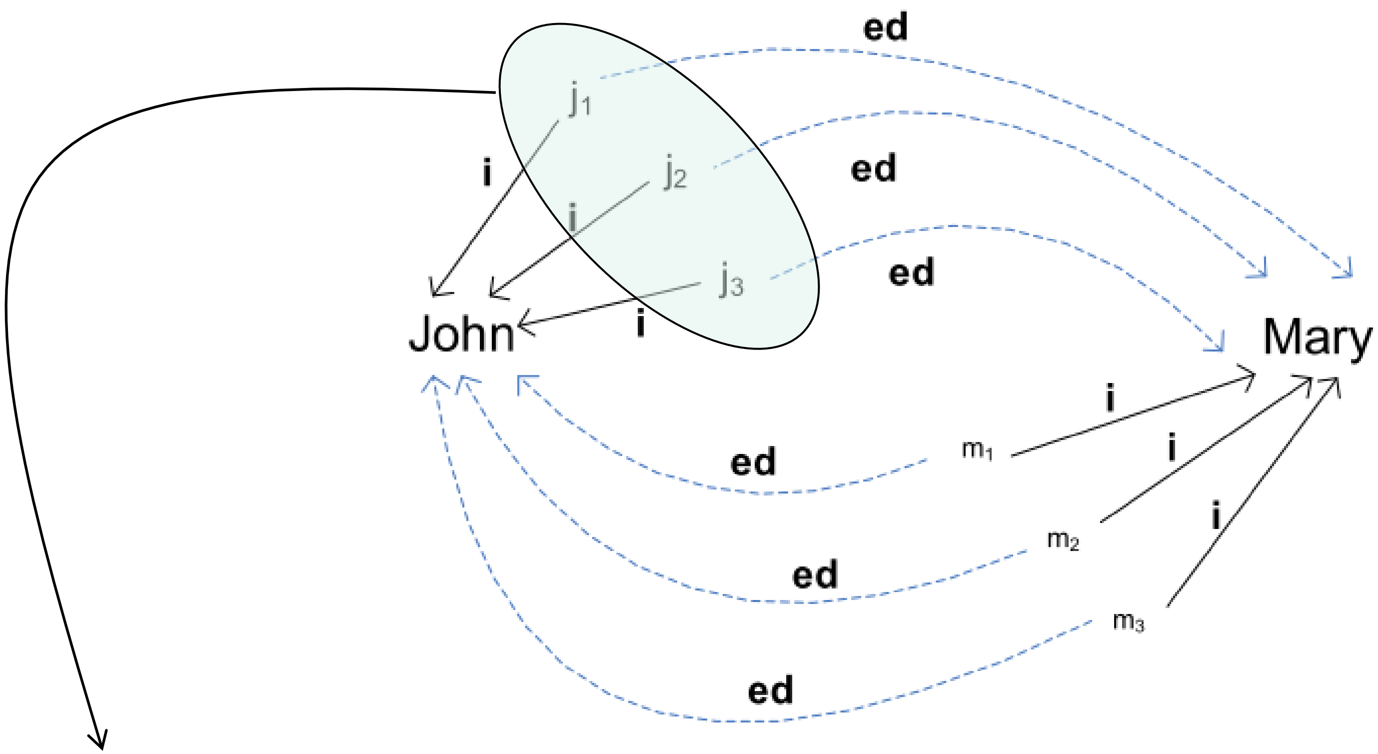
1. Now, suppose these are all the properties that John acquire by virtue of being married to Mary (e.g., all rights and responsibilities towards Mary)

Externally Dependent Aspect



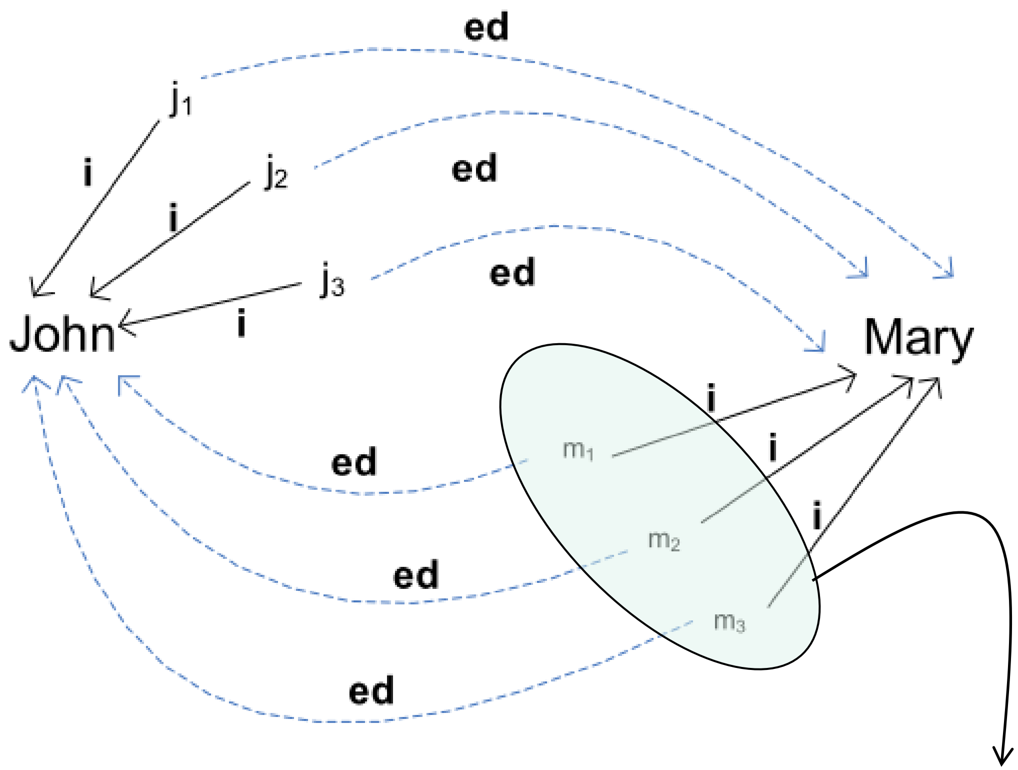
1. Now, suppose these are all the properties that John acquire by virtue of being married to Mary (e.g., all rights and responsibilities towards Mary)
2. These properties are acquired by John due to the happening of a **founding event** (e.g., the wedding, the signing of a social contract)

Externally Dependent Aspect



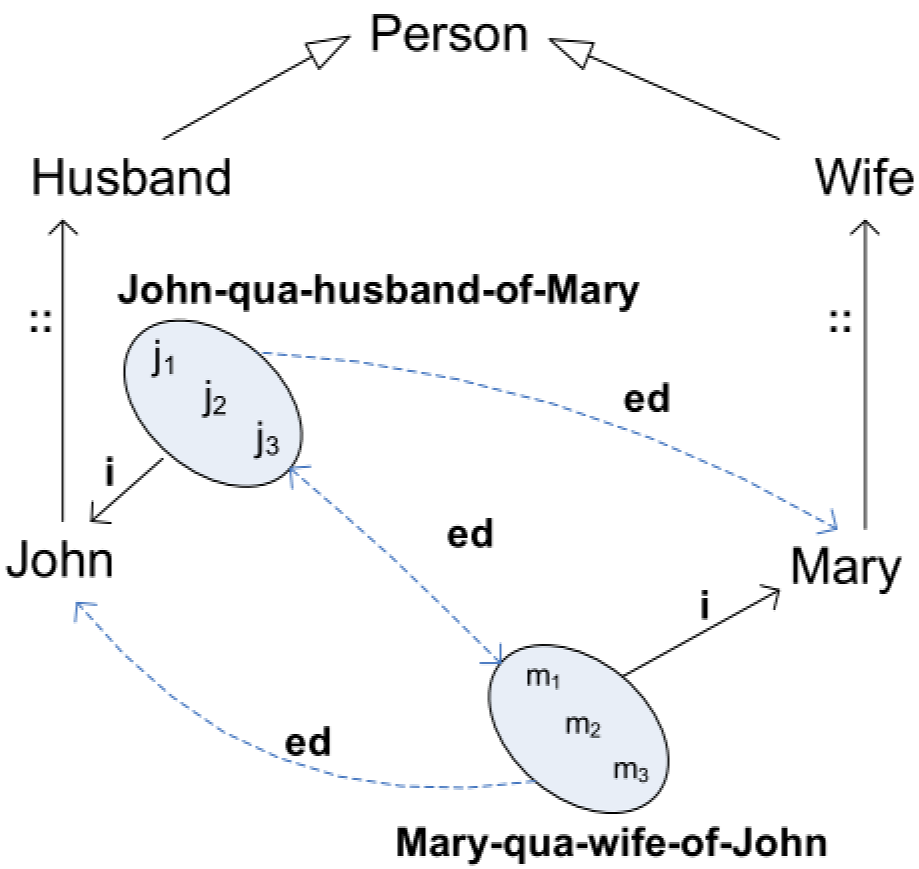
1. Now, suppose these are all the properties that John acquire by virtue of being married to Mary (e.g., all rights and responsibilities towards Mary)
2. These properties are acquired by John due to the happening of a **founding event** (e.g., the wedding, the signing of a social contract)
3. These are properties of John (inhering in John) but which are also existentially dependent on Mary

Externally Dependent Aspect



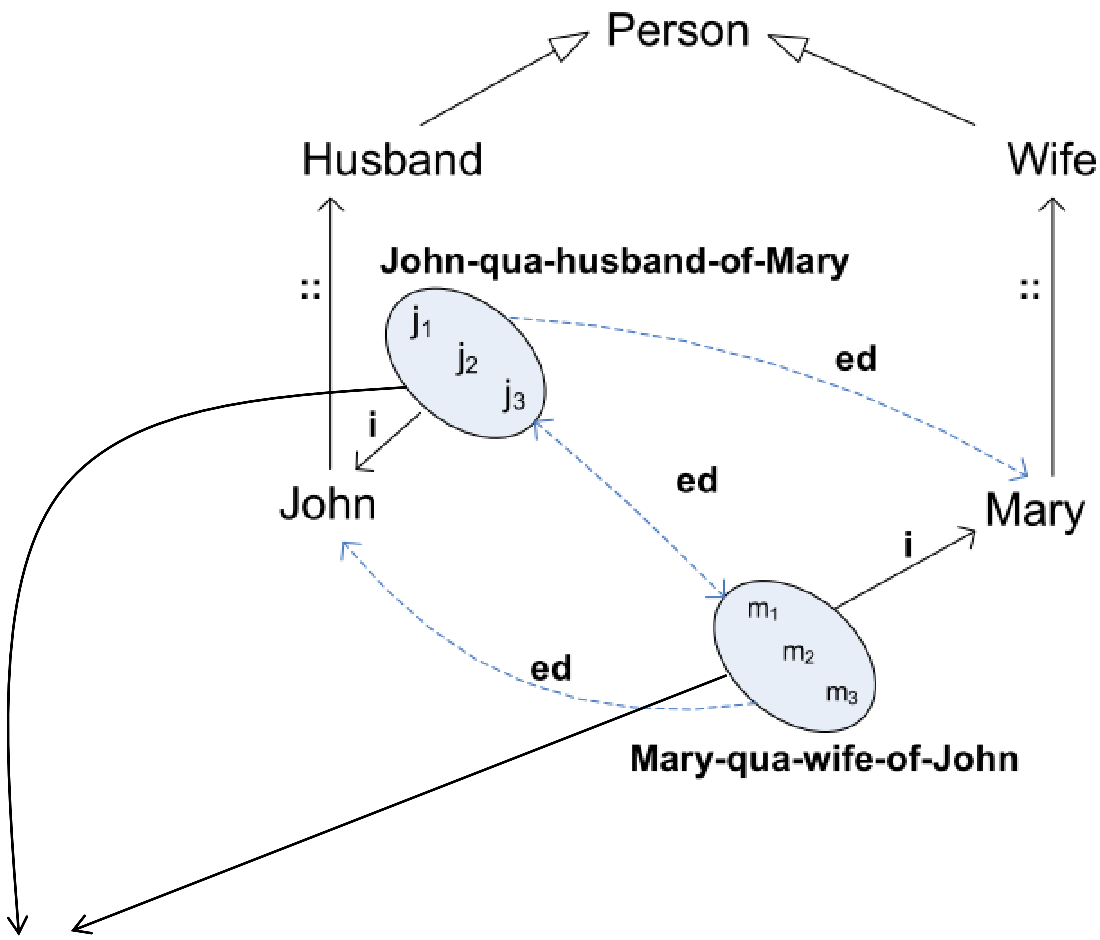
1. Analogously, we have all the properties that Mary acquire by virtue of being married to John (e.g., all rights and responsibilities towards John)
2. These properties are again acquired by Mary due to the happening of a **founding event** (e.g., the wedding, the signing of a social contract)
3. These are properties of Mary (inhering in Mary) but which are also existentially dependent on John

Externally Dependent Aspects



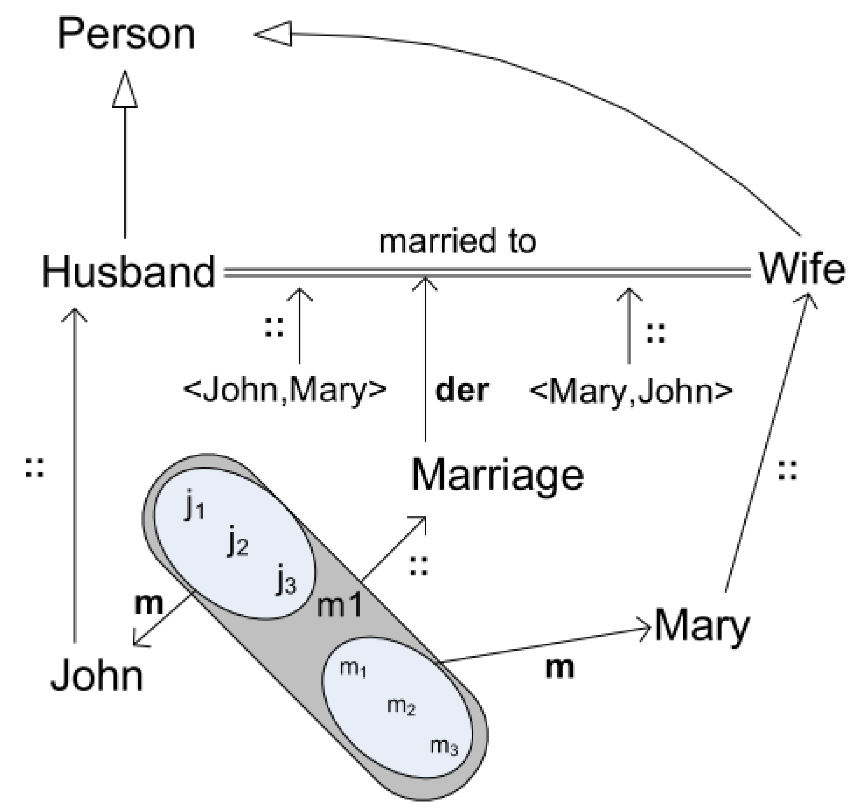
1. The entity which is the aggregation of all the properties of an entity which share the same founding event and which are externally dependent on the same entity is named a **qua individual**

Externally Dependent Aspects



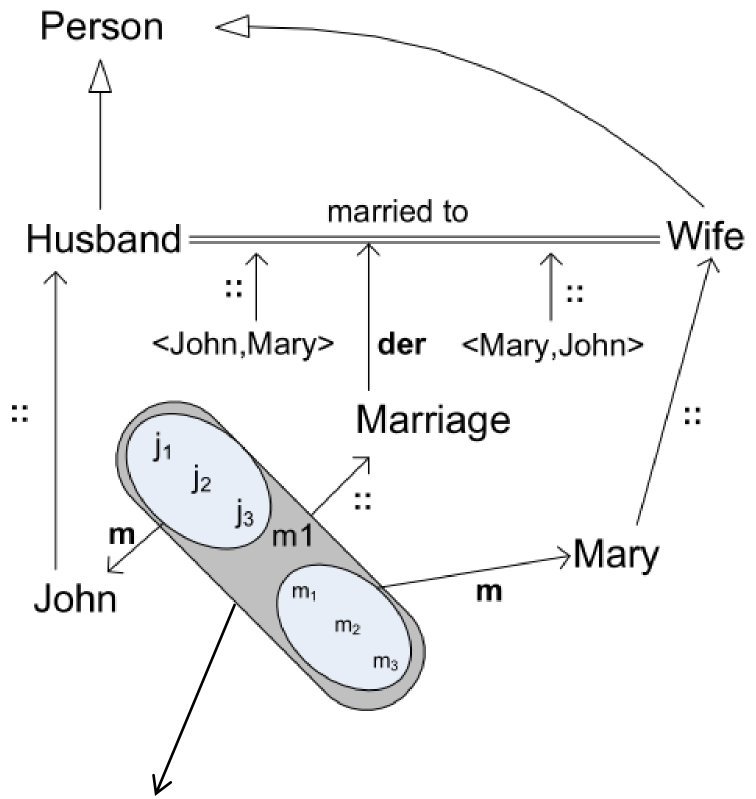
2. **Qua individuals** are special types of Externally Dependent Modes
3. They constitute and **aspectual slice of an entity**, representing the view of an entity in the context of a material relation, i.e., they represent the aggregation of properties that an entity has in the scope of a relation

Relators



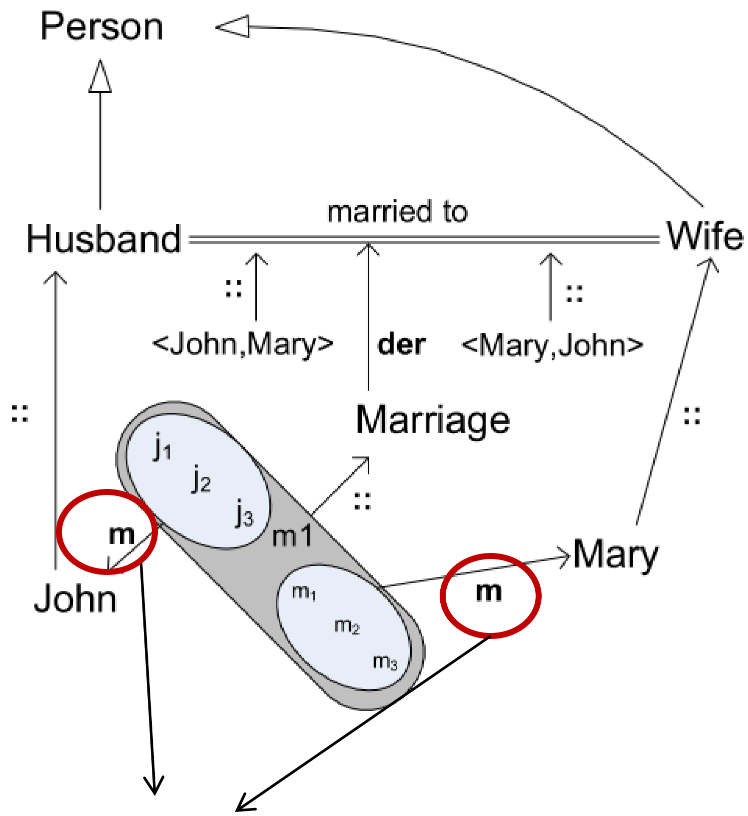
Now, we can create an entity which is the aggregation of all qua individuals that share the same founding event. We name this entity a relator

Relators



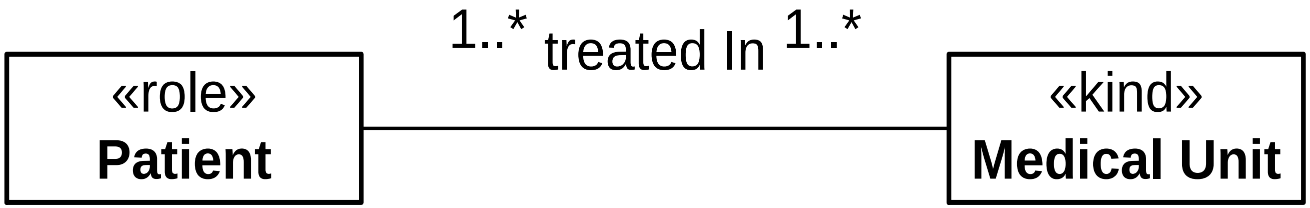
1. In this case, the relator Marriage is the entity which is *existentially dependent* on John and Mary, thus, connecting the two.
2. Marriage represents the aggregation of all properties that John and Mary have towards each other (e.g., rights and responsabilities) by virtue of the establishment of that relation, or more precisely, by virtue of the same founding event (e.g., wedding, signing of social contract)

Relators



1. We define a relation of mediation between a relator and the entities it connects.
2. Mediation is a type of *existential dependence relation* (a form of non-functional inherence)

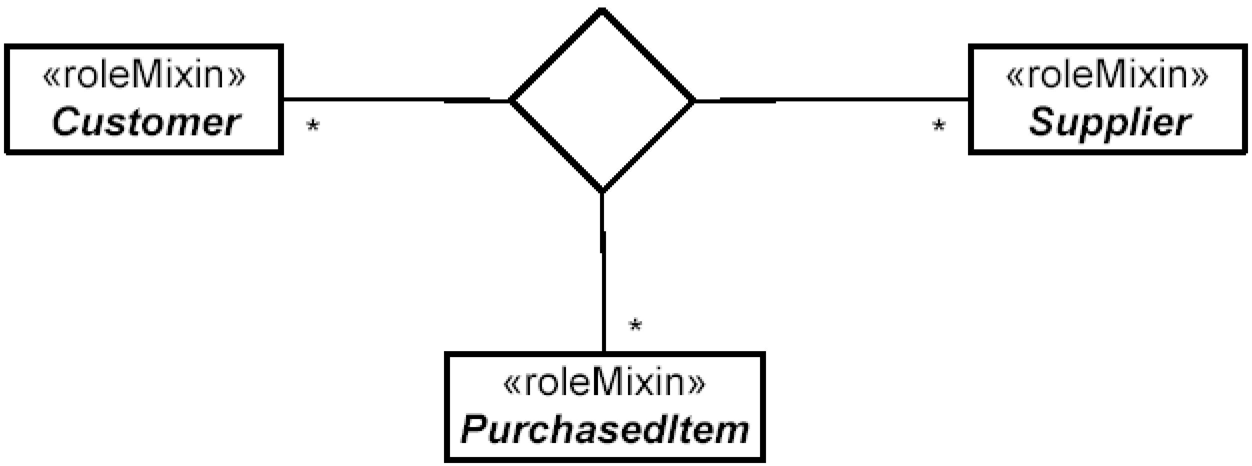
Material Relations



Material Relations

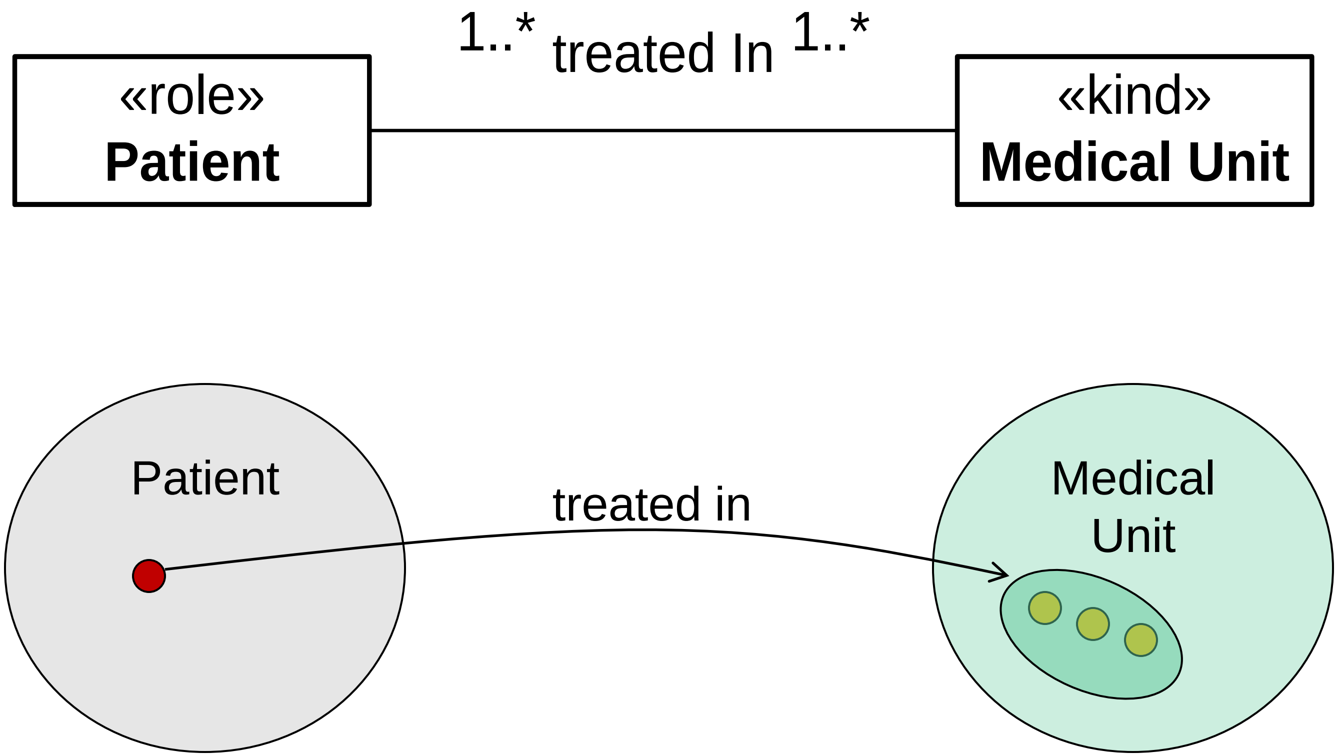
- How are these cardinality constraints to be interpreted ?
  - In a treatment, a patient is treated by several medical units, and a patient can participate in many treatments
  - In a treatment, a patient is treated by several medical units, but a patient can only participate in one treatment
  - In a treatment, several patients can be treated by one medical unit, and a medical unit can participate in many treatments
  - In a treatment, a patient is treated by one medical unit, and a patient can participate in many treatments
  - ...

- This problem is even worse in n-ary association(with n > 2)

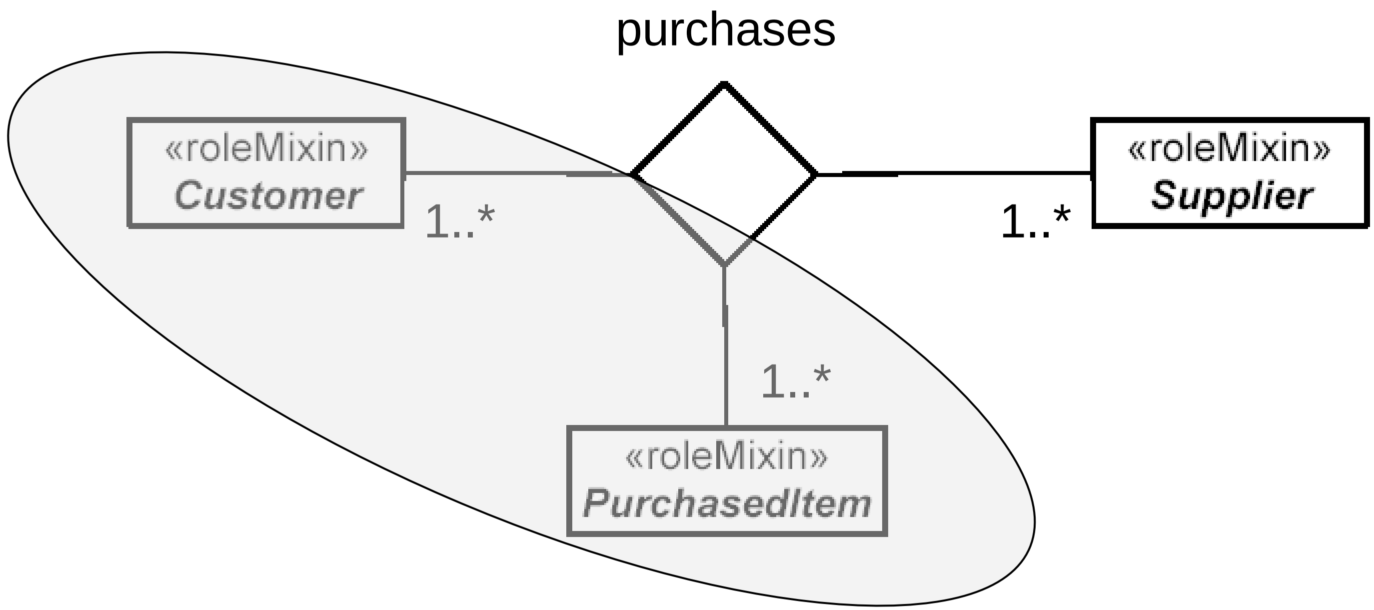


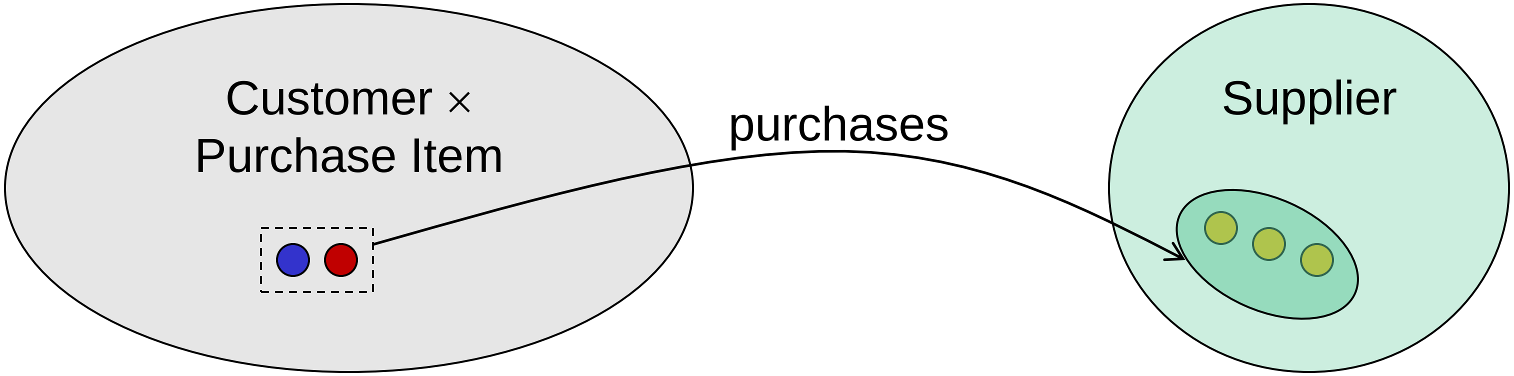
1. In a given purchase, a Customer participates by buying many items from many Suppliers and a customer can participate in several purchases;
2. In a given purchase, many Customers participate by buying many items from many Suppliers, and a customer can participate in only one purchase;
3. In given purchase, a Customer participates by buying many items from a Supplier, and a customer can participate in several purchases;
4. In given purchase, many Customers participate by buying many items from a Supplier, and a customer can participate in several purchases
5. …

Extensional Semantics of Cardinality Constraints



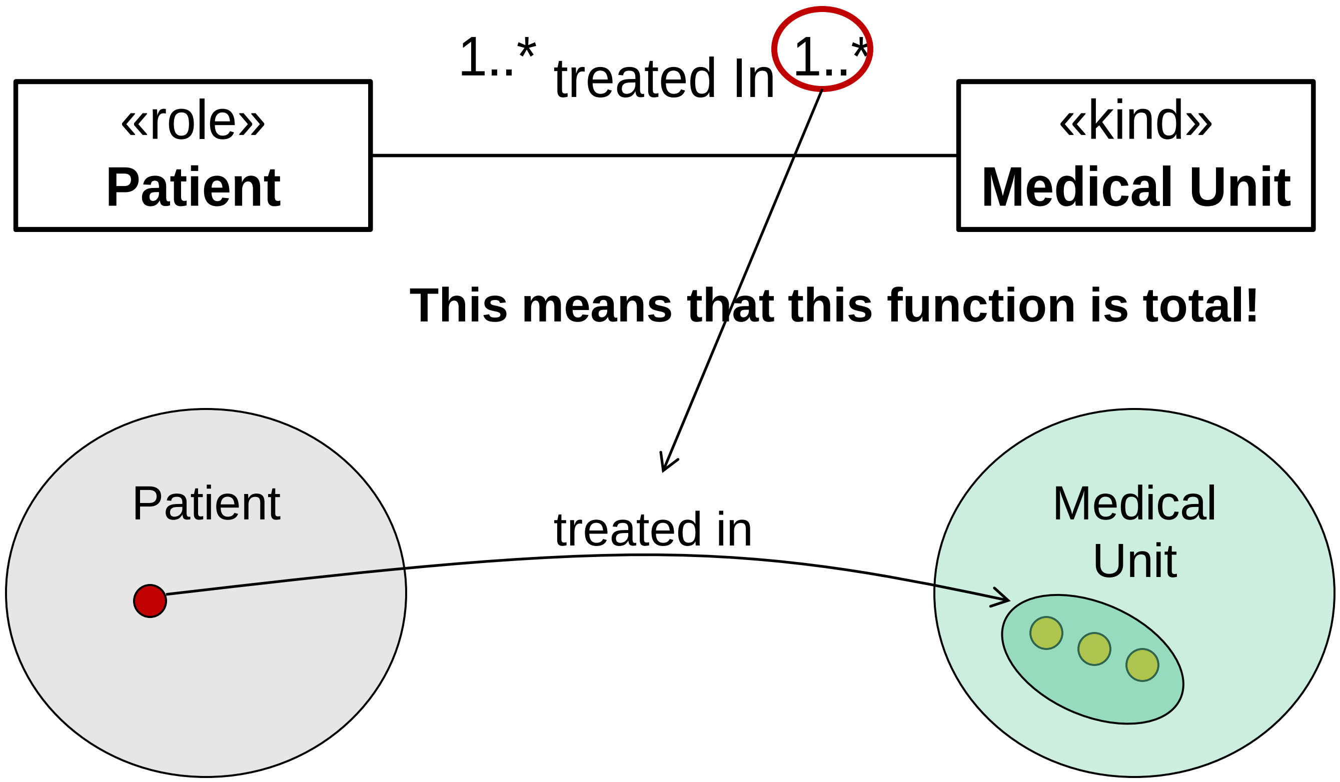
treated in: Patient $\rightarrow \wp$(Medical Unit)



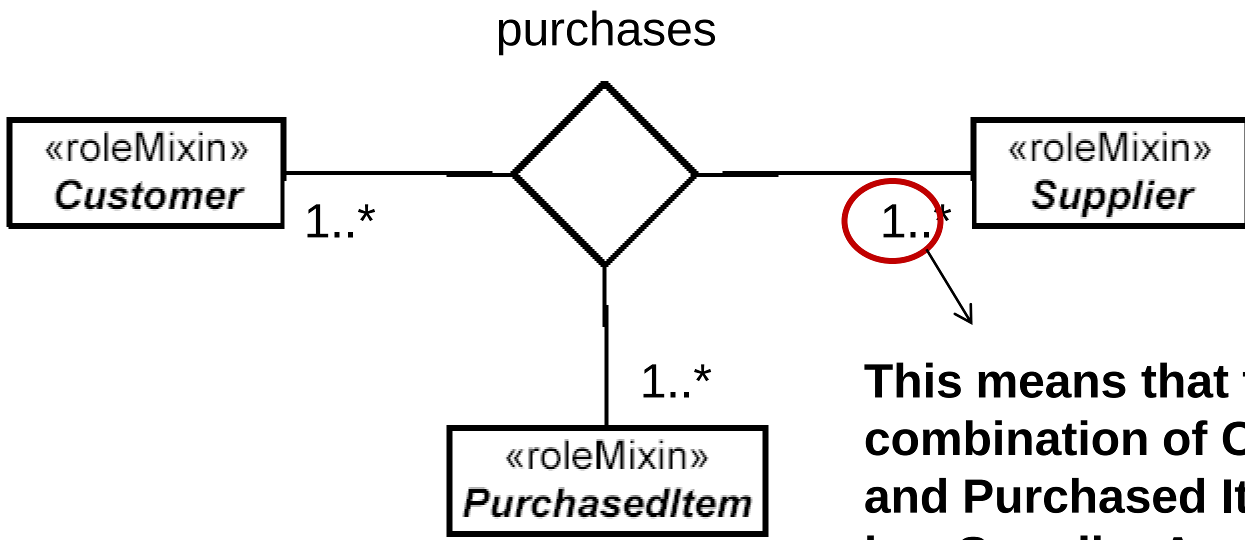


purchases: Customer × Purchase Item $\rightarrow \wp$(Supplier)

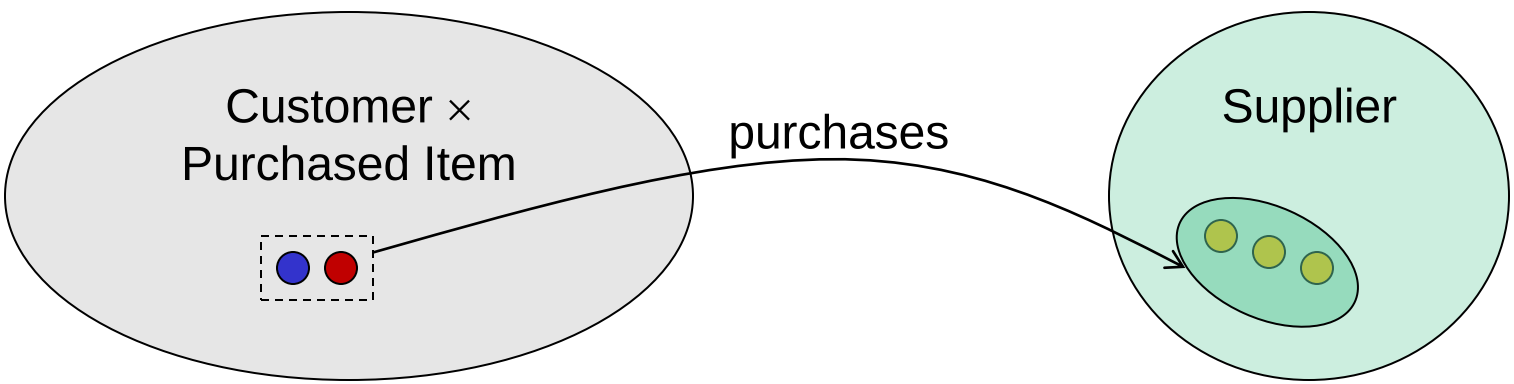
Extensional Semantics of Cardinality Constraints



treated in: Patient $\rightarrow \wp$(Medical Unit)



This means that for any combination of Customer and Purchased Item there is a Supplier Associated! Even if the Customer did not buy that Item!

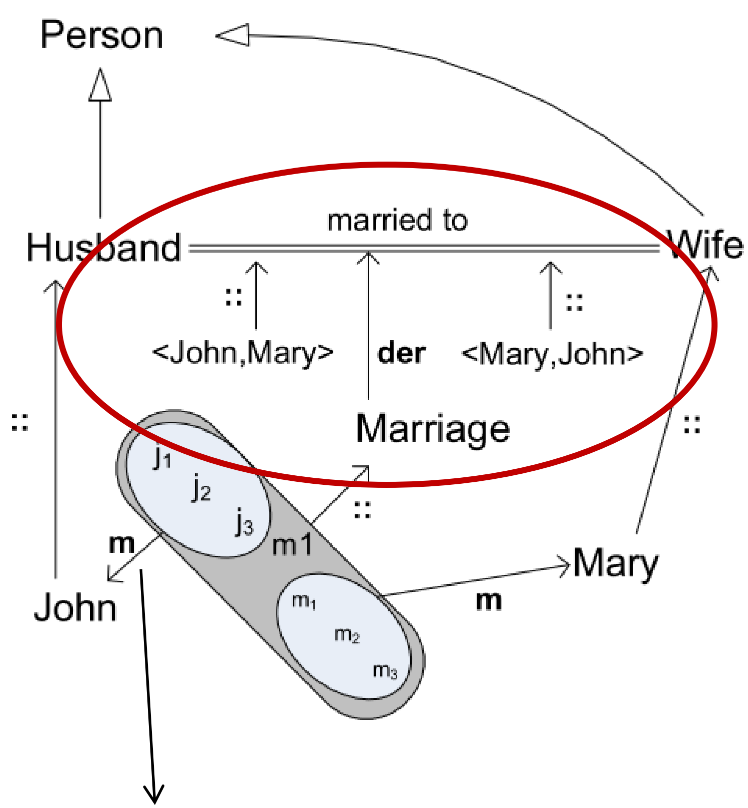


purchases: Customer $\times$ Purchased Item $\rightarrow \wp$(Supplier)

N-Ary Relations

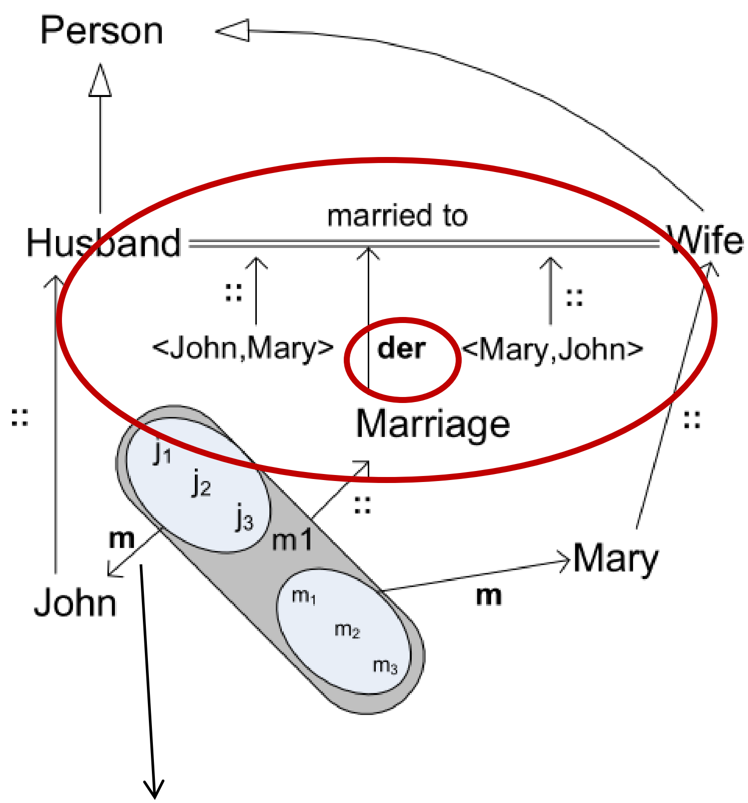
- In summary, for practically all n-ary relations, the minimum cardinality constraints will be equal to zero.
- Since this is the same as imposing no constraint, this limitation renders the specification of minimum cardinality constraints useless in these representation systems

Relators



1. In fact the material relation can be completely derived from the relator and the corresponding mediation relations

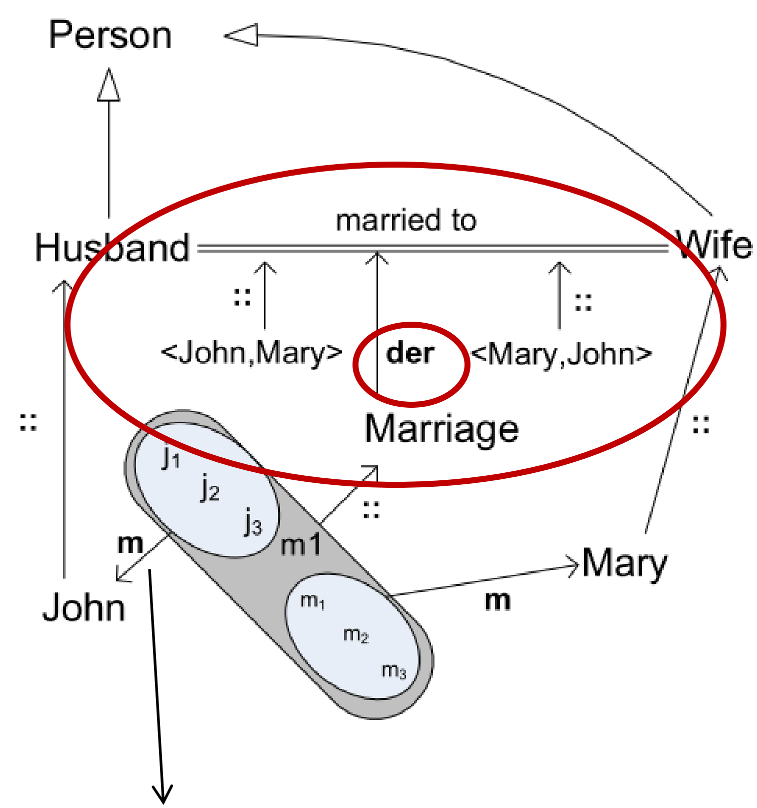
Relators



2. We say that that the relator typeT_R induces a material relation R or that R is derived from T_R (symbolized as *der(R,*T_R*)*) if and only if

$$\mathbf{R(x,y) \leftrightarrow \exists r\ r::T_R \wedge m(r,x) \wedge m(r,y)}$$

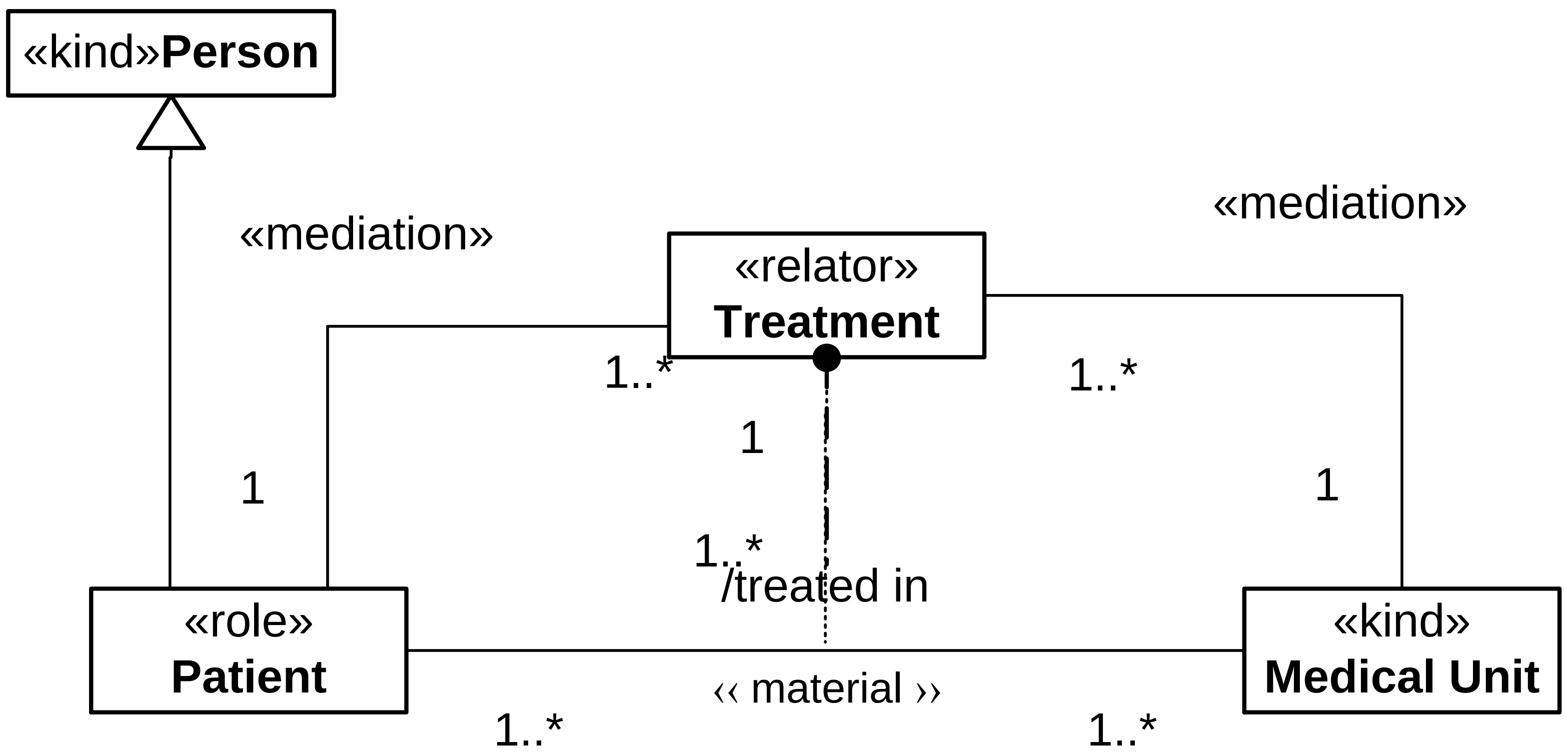
Relators



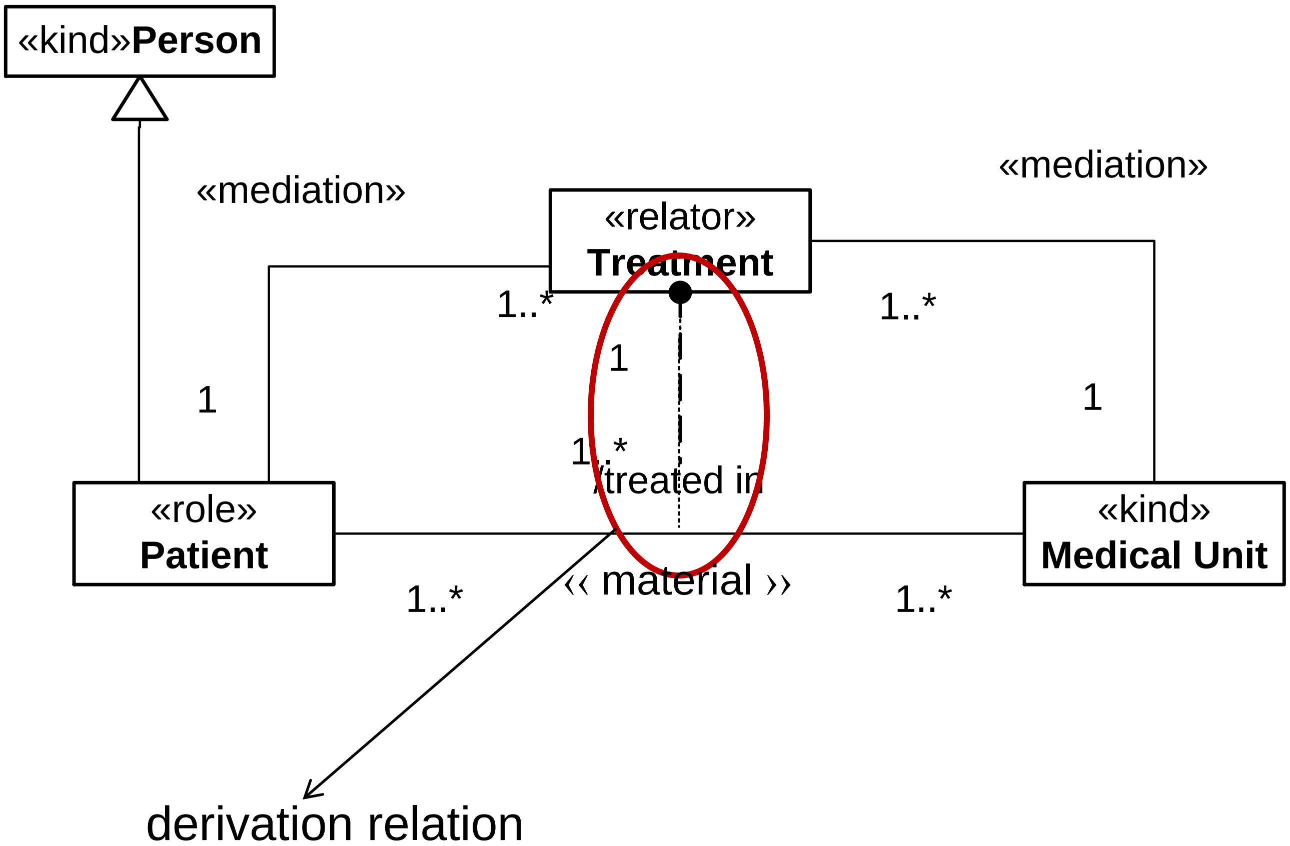
3. In this case, we have that *der(married to,Marriage)*) and than

Married to(x,y) ↔ ∃m::Marriage ∧ m(m,x) ∧ m(m,y)

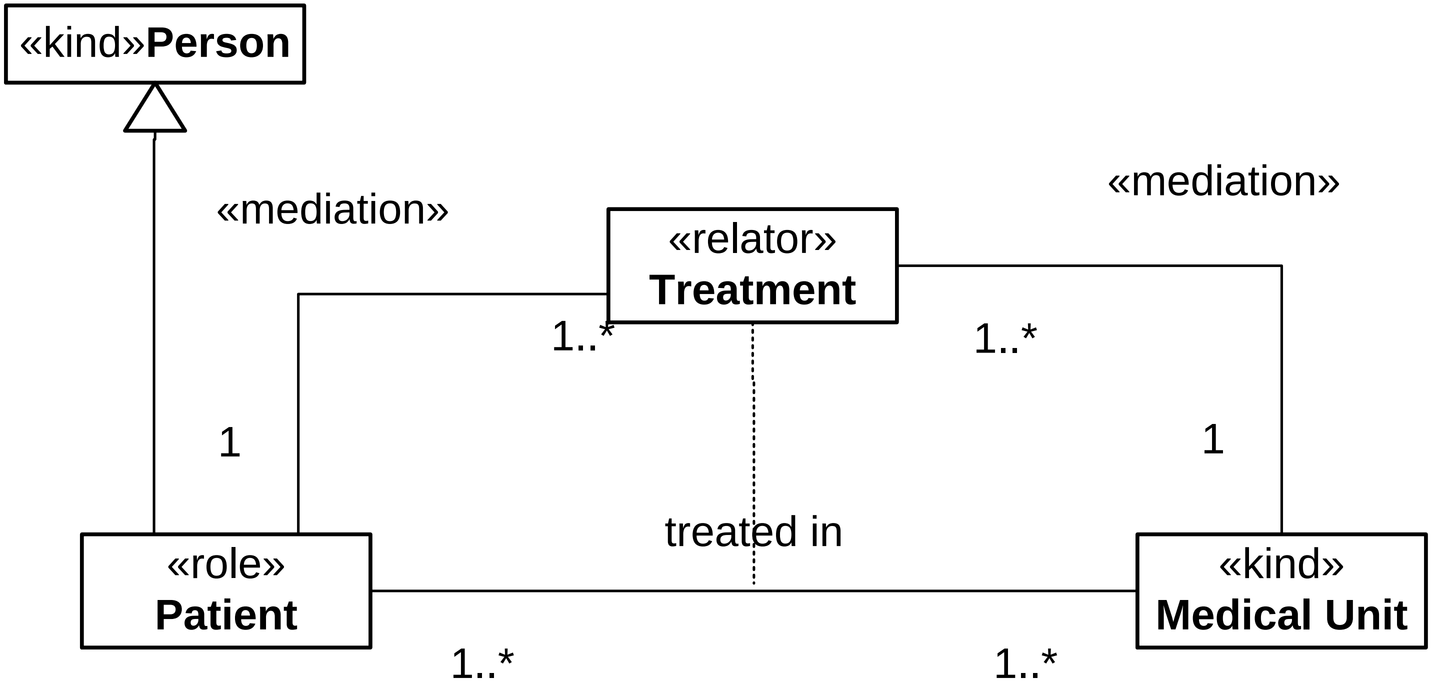
Relators and Derived Material Relations



Relators and Derived Material Relations

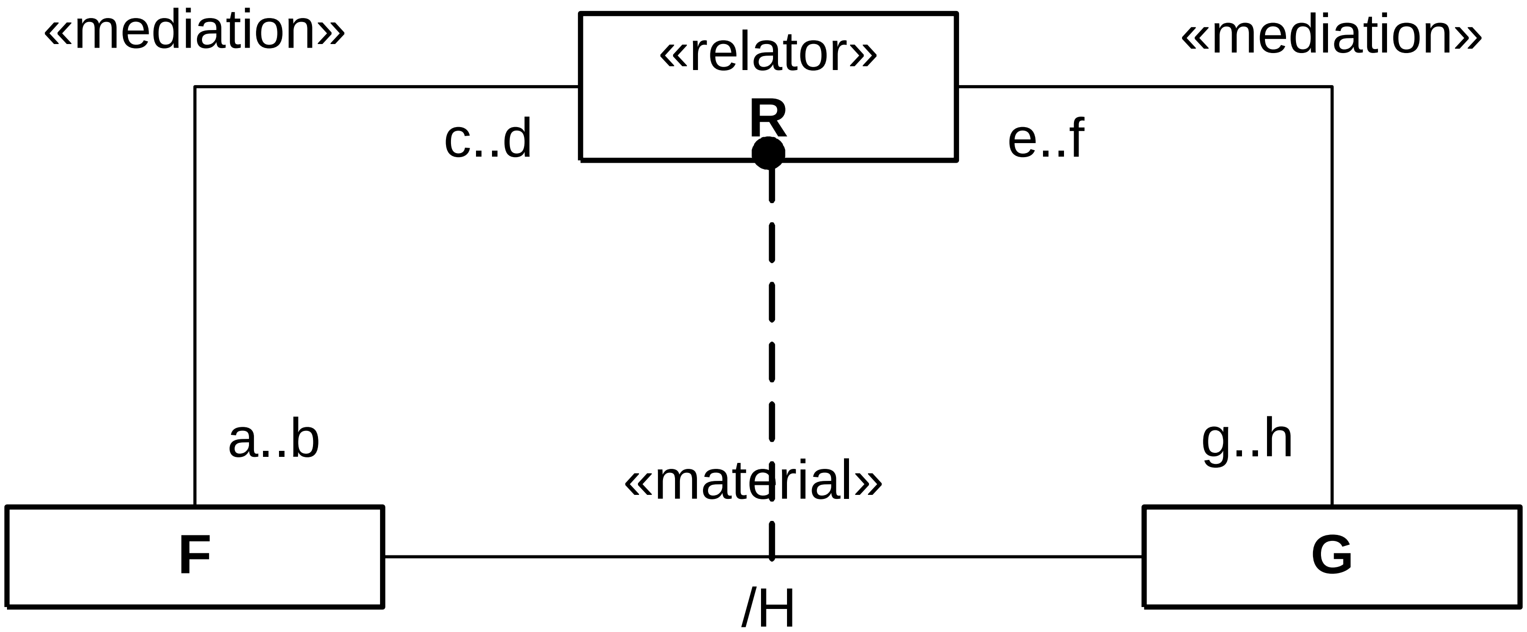


Not the same as Association Classes…

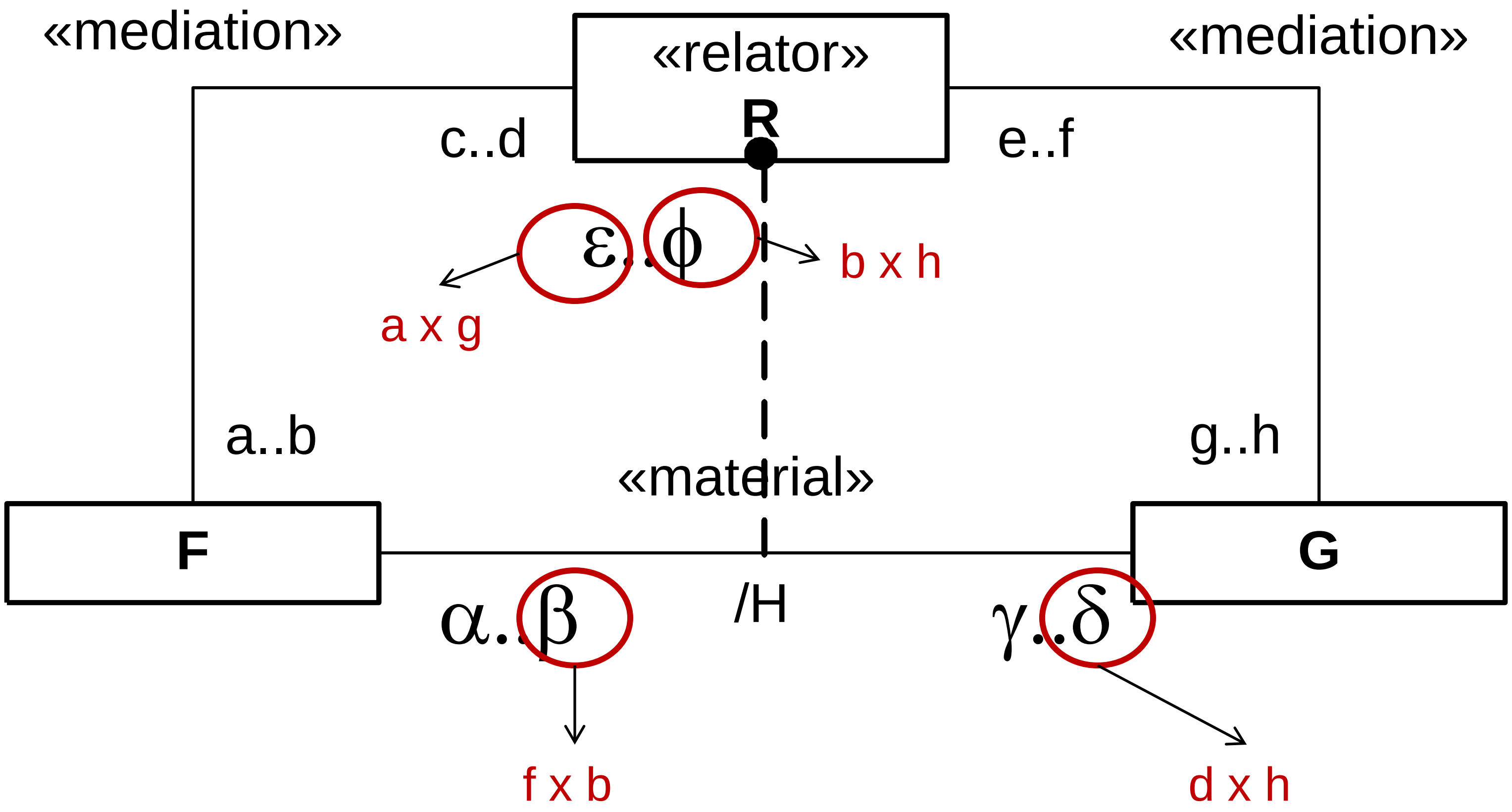


In the case of association classes, the relation and the association class are one and the same entity! As a consequence, we cannot have two different pairs of Patient and Medical Unit which are mediated by exactly the same Treatment!

Deriving Cardinality Constraints



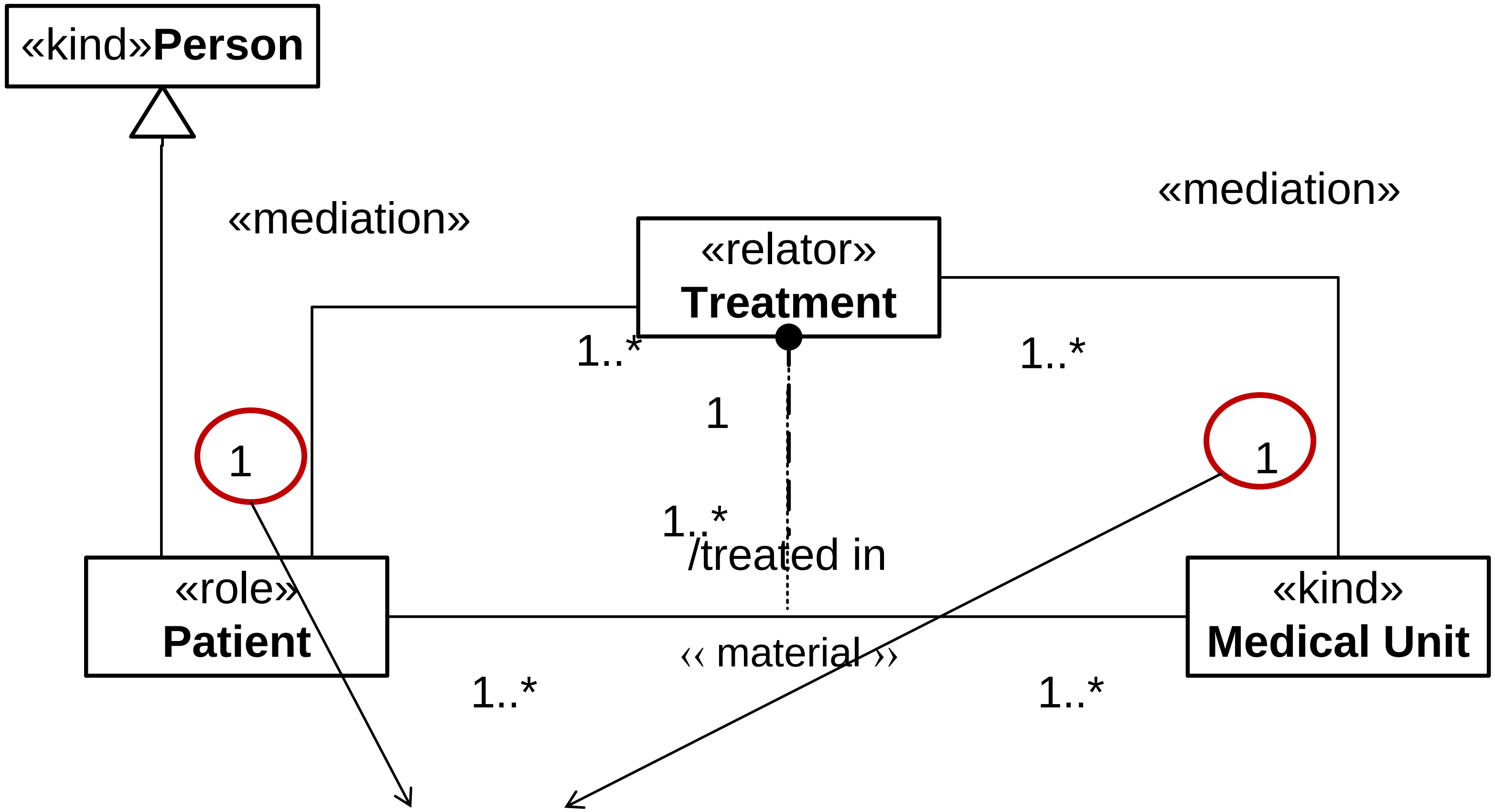
Deriving Cardinality Constraints



Material Relations

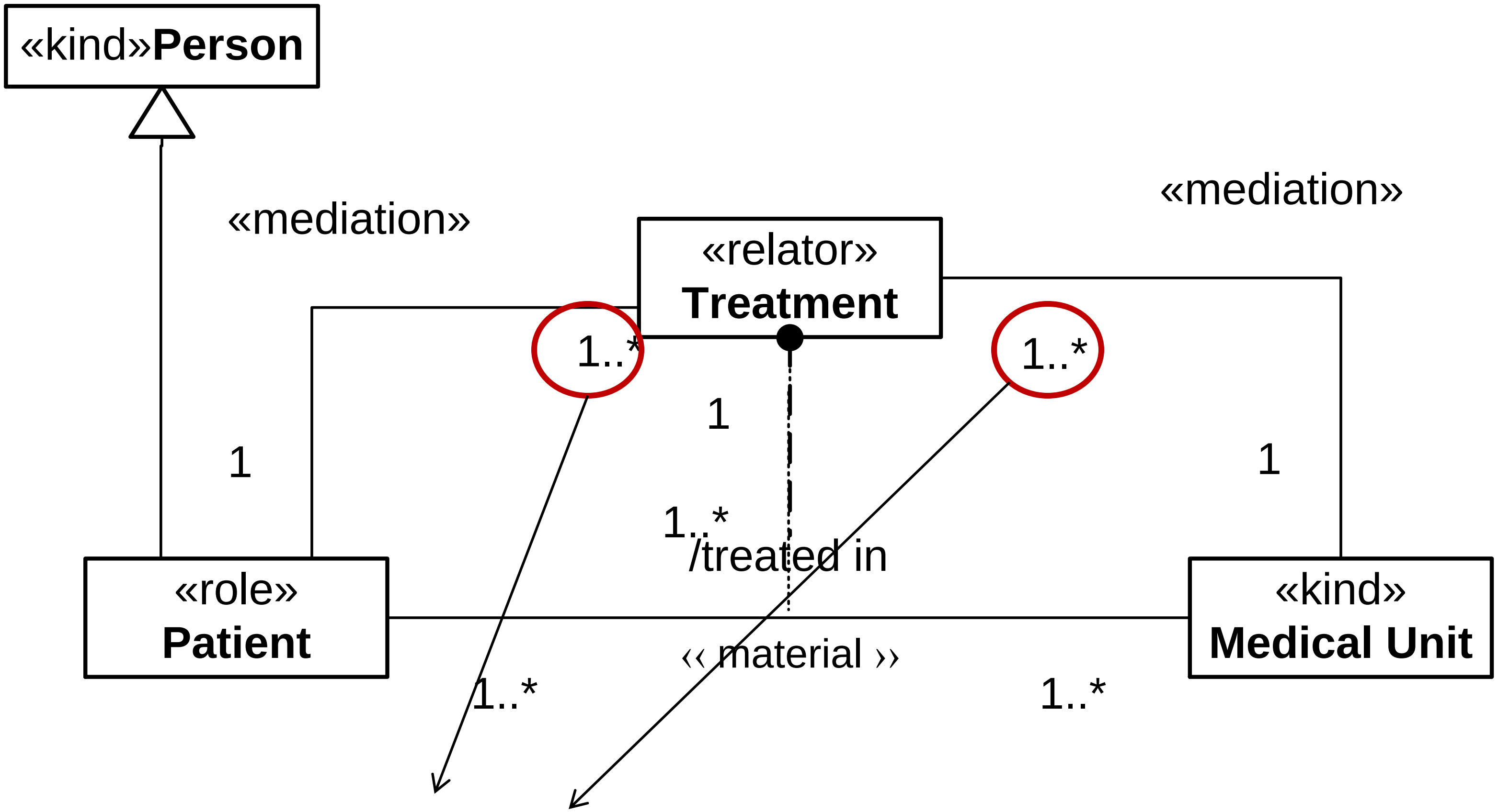
- As seen before from a relator and mediation relation we can derive several material relations
- Asides from all the benefits previously mentioned, perhaps the most important contribution of explicitly considering relations is to force the modeler to answer the fundamental question of what is *truthmaker* of that relation. In other words, what that relation really means!
- Making the relator explicit it is to make the semantics of the (material) relation explicit. Notice that It is very easy for people to hide domain knowledge under a predicate, thus, maintaining that knowledge tacit!

The Problem of Collapsing Cardinality Constraints



single-tuple cardinality constraints

The Problem of Collapsing Cardinality Constraints



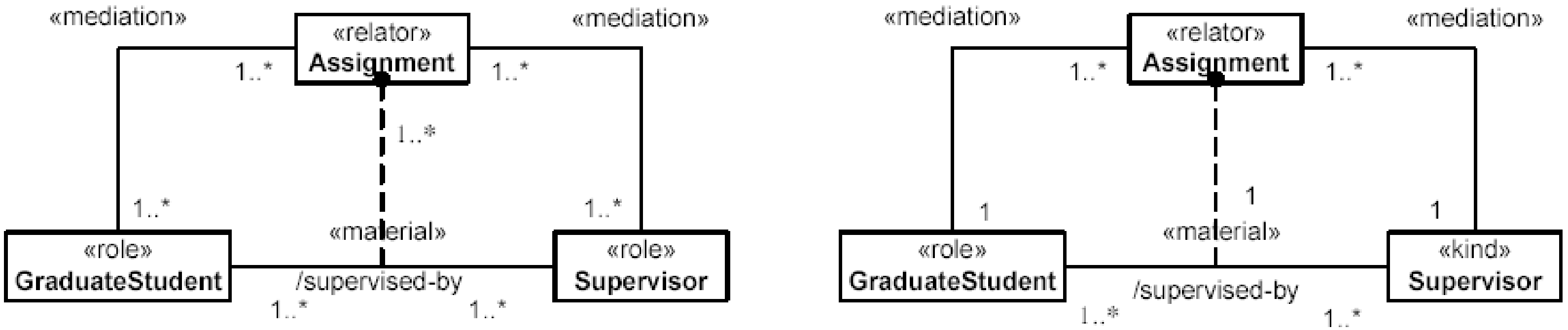
multiple-tuple cardinality constraints

Material Relations

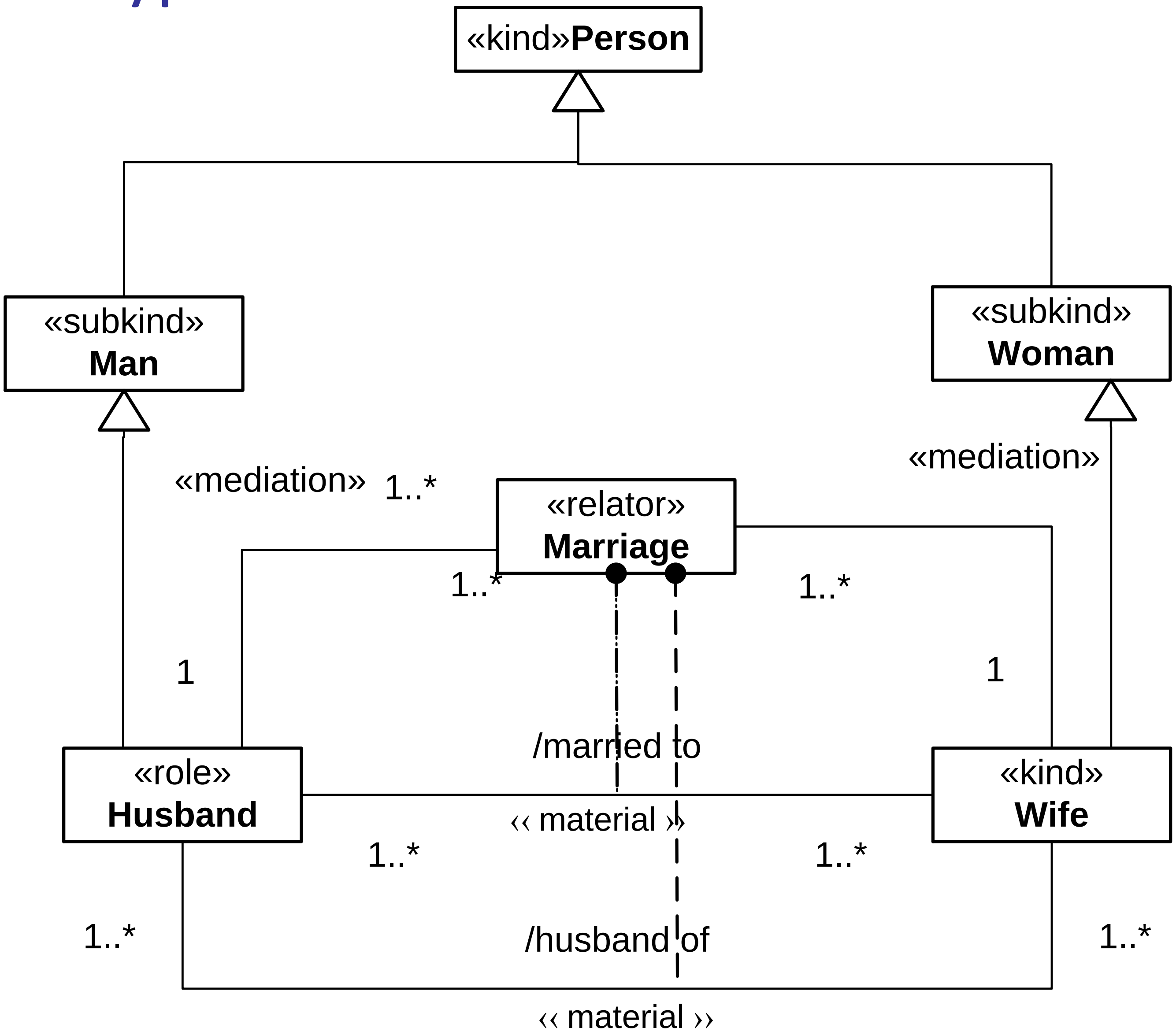
- Most existing conceptual modeling notations collapse single-tuple and multiple-tuple cardinality constraints in one single representation
- Notice that the problem of collapsing cardinality constraints only affects material relations and always affects material relations! (When there seem to be no ambiguity, it is because there is one interpretation among the many possible, which is more salient!)
- That is why it is so important to distinguish between formal and material relations

Material Relations

- Yet another example:
  - Modeling that a graduate student have one or more supervisors and a supervisor can supervise one or more students

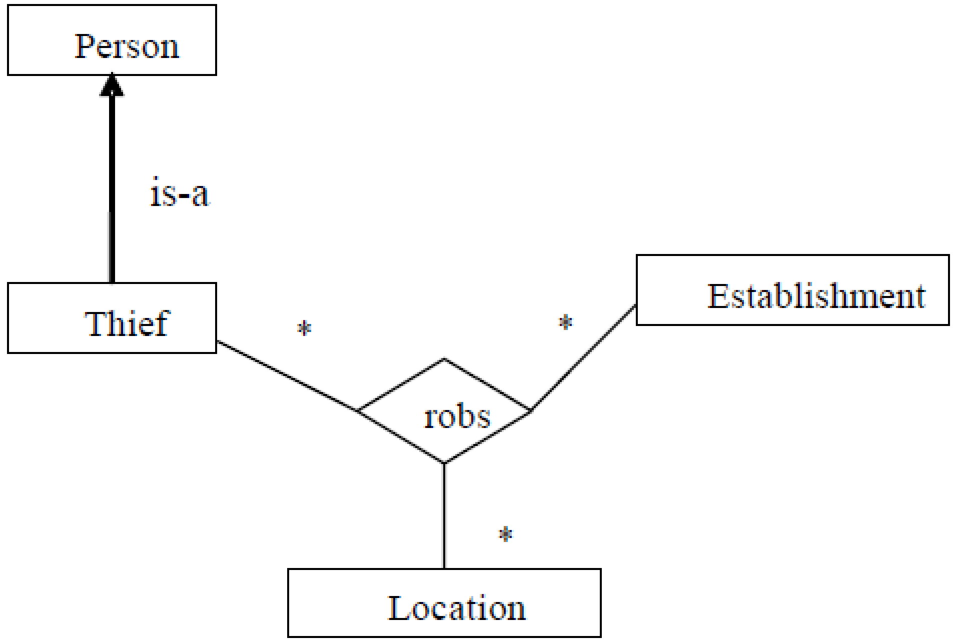


Material Relations derived from the same Relator Type



An extra linguistic-based example

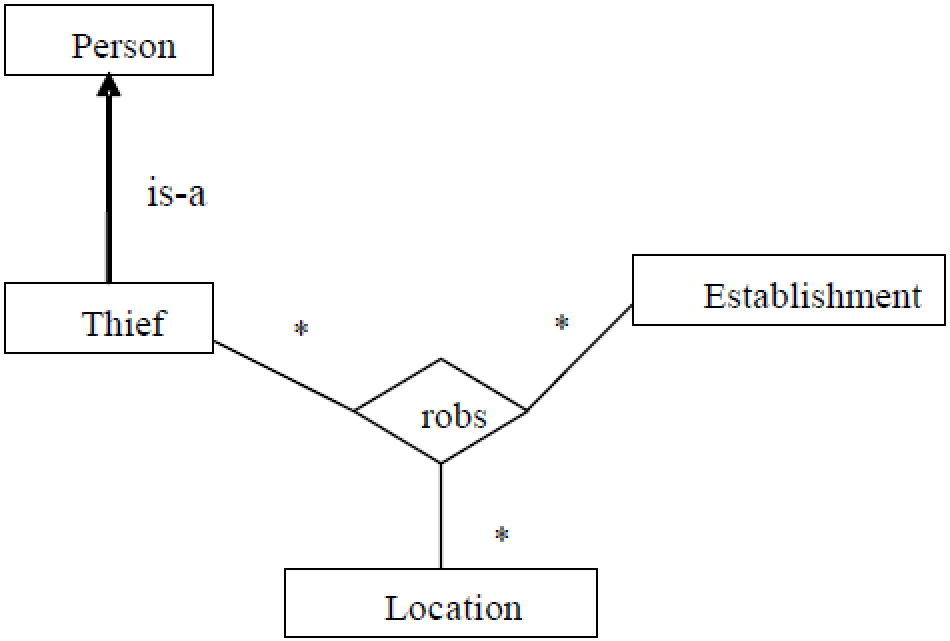
- 1. Sarah robs the 7-11 in New York.
- 2. Adam robs in Washington in February.

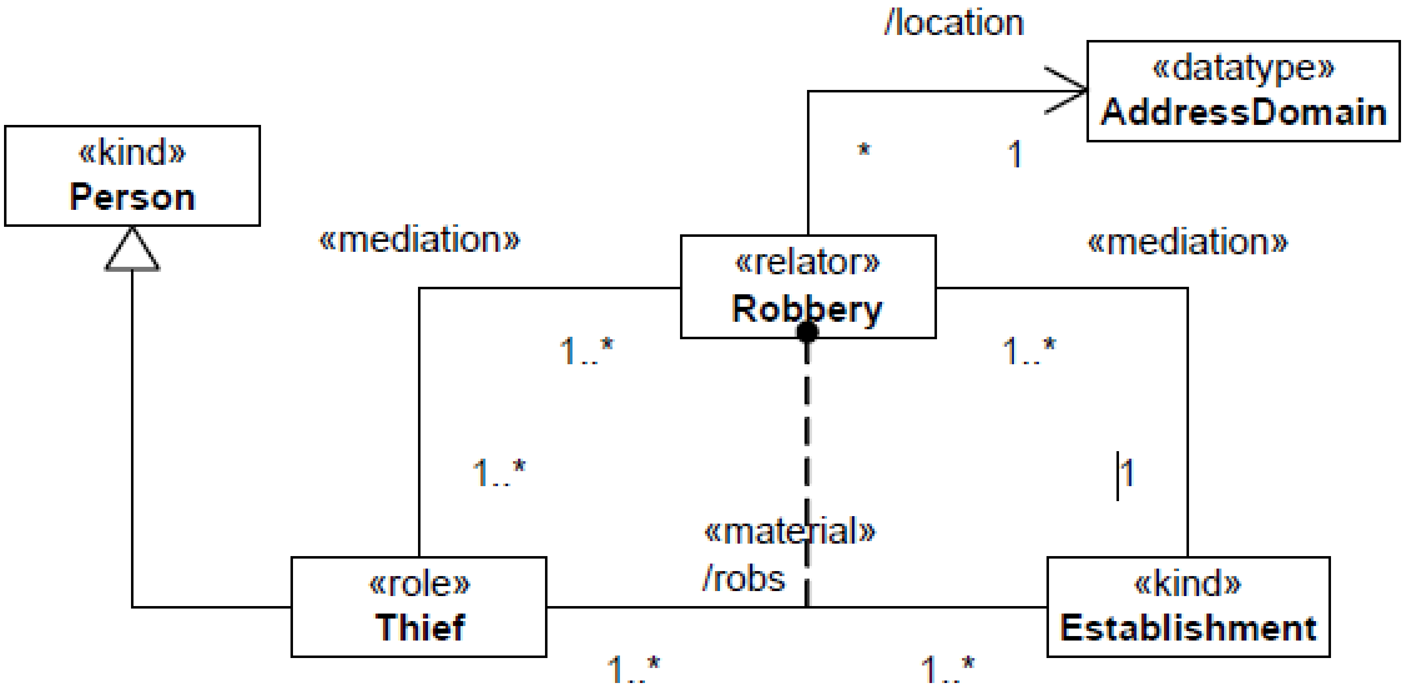


An extra linguistic-based example

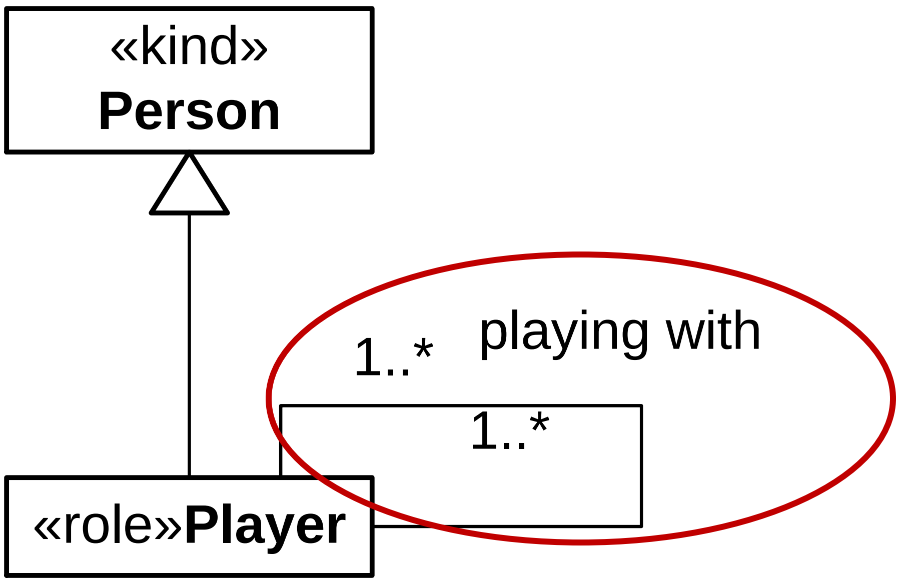
- 1. Sarah robs the 7-11 in New York.
- 2. Adam robs in Washington in February.

Much closer Relation!



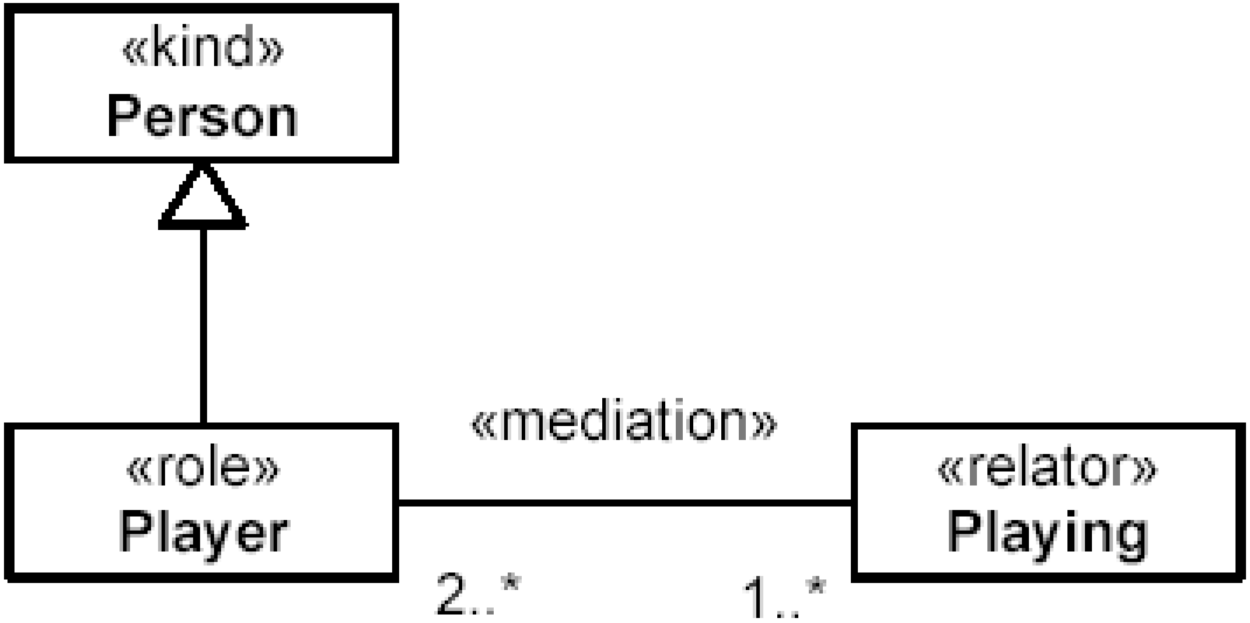

/location
«datatype»
AddressDomain
«kind»
Person
*
1
«mediation»
«relator»
Robbery
«mediation»
1..*
1..*
1..*
1
«material»
/robs
«role»
Thief
«kind»
Establishment
1..*
1..*

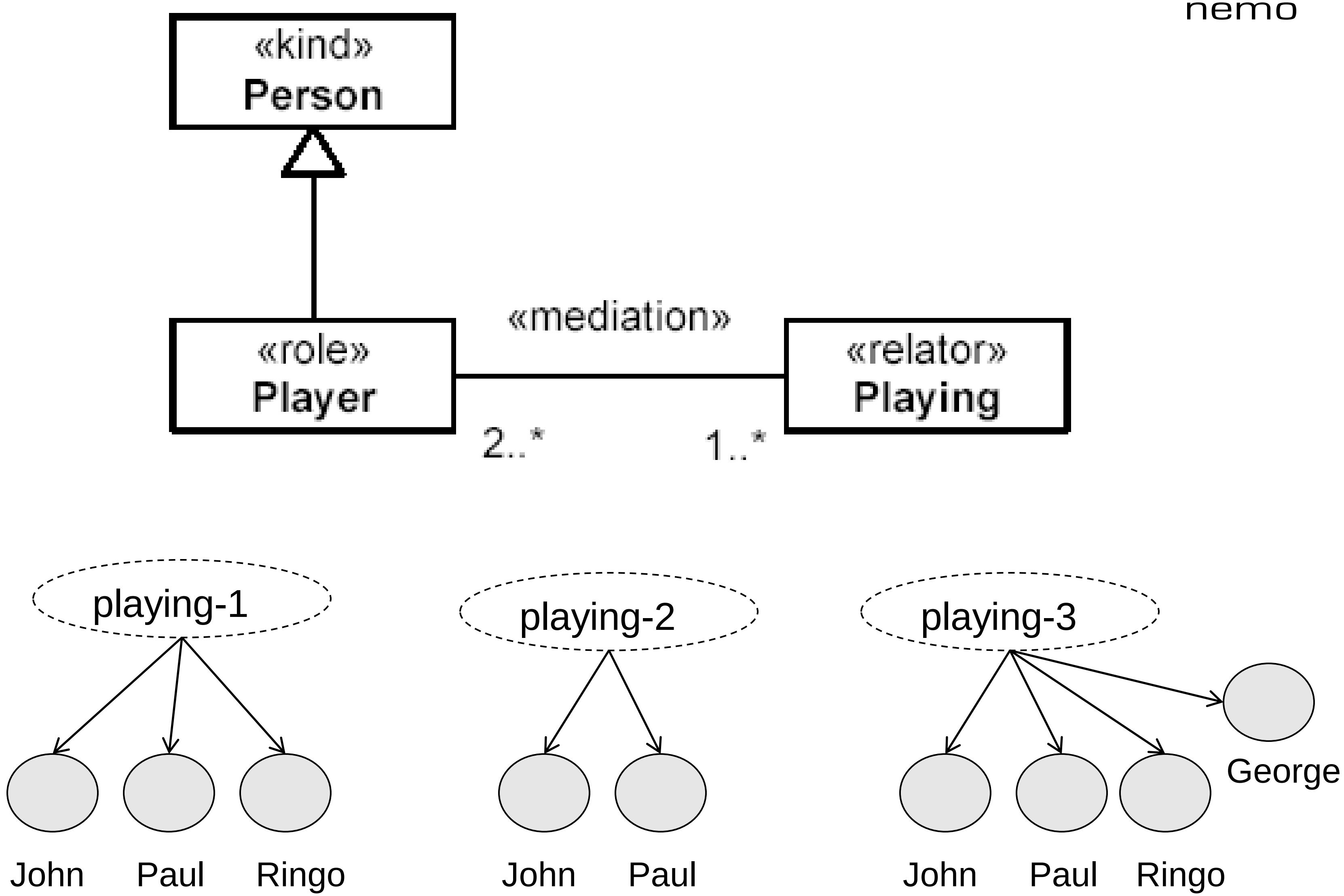
Anadic Relations



Notice that this does not capture the semantics of this anadic relation. The playing with relation here is binary, not anadic! In every instance of the relation, there is a pair of people playing, despite the fact that the same individual can participate in several of those pairs!

Anadic Relations




«kind»
Person
«role»
Player
«mediation»
«relator»
Playing
2..*
1..*
playing-1
playing-2
playing-3
George
John
Paul
Ringo
John
Paul
John
Paul
Ringo

nemo

gguizzardi@inf.ufes.br